ARTICLE IN PRESS

Neurocomputing ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Fuzzy generalized projective synchronization of incommensurate fractional-order chaotic systems

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ARTICLE INFO

Article history:
Received 17 April 2015
Received in revised form
24 July 2015
Accepted 2 August 2015
Communicated by Shaocheng Tong

Keywords:
Adaptive fuzzy control
Incommensurate fractional-order systems
Uncertain chaotic systems
Generalized projective synchronization

ABSTRACT

This paper proposes a novel fuzzy adaptive controller for achieving an appropriate generalized projective synchronization (GPS) of two incommensurate fractional-order chaotic systems. The master system and the slave system, considered here, are assumed to be with non-identical structure, external dynamical disturbances, uncertain models and distinct fractional-orders. The adaptive fuzzy systems are used for estimating some unknown nonlinear functions. A Lyapunov approach is adopted for deriving the parameter adaptation laws and proving the stability of the closed-loop system. Under some mild assumptions, the proposed controller can guarantee all the signals in the closed-loop system remain bounded and the underlying synchronization errors asymptotically converge towards a small of neighborhood of the origin. Finally, some numerical experiment results are presented to illustrate the effectiveness of the proposed synchronization scheme.

1. Introduction

Fractional calculus is an area of mathematics that handles with differentiation and integration of arbitrary (non-integer) orders. In recent years, the fractional calculus has been studied with increasing interest from physicians, chemists, and engineers. In fact, it was found that various systems in interdisciplinary fields can be accurately modeled by fractional-order differential equations such as [1,2]: viscoelastic systems, dielectric polarization, electrode–electrolyte polarization, finance systems, electromagnetic waves, heat diffusion systems, batteries, neurons, and so on. That is to say, fractional derivatives give a superb instrument for an accurate description of memory and heredity features of many material and processes.

Chaotic systems are nonlinear and deterministic rather than probabilistic, and they are characterized by the self similarity of the strange attractor and unusual sensitivity to initial conditions quantified by fractal dimension and the existence of a positive Lyapunov exponent, respectively. However, a hyperchaotic system is characterized as a chaotic system but with more than one positive Lyapunov exponents. Recently, many literatures shown that some fractional-order systems can behave chaotically or hyperchaotically, e.g. fractional-order Lorenz system [3], fractional-order Rössler system [4], fractional-order Lü system [5], fractional-order Arneodo system [6], to name a few.

Synchronization problem consists in designing a system (slave system) whose behavior mimics another one (master system). The latter drives the slave system via the transmitted signals. In the literature, various types of the chaos synchronization have been revealed, such as complete synchronization (CS) [7–9], phase synchronization (PHS) [10,11], lag synchronization (LS) [12,13], generalized synchronization (GS) [14], generalized projective synchronization (GPS) [15–17], and so on. However, all these synchronization methods focus on integer-order chaotic systems, which is a very special case of the non-integer-order (i.e. fractional-order) chaotic systems. In addition, it has been assumed in [7–17] that models of the chaotic systems are almost known. Therefore, it is very interesting to extend these fundamental results to uncertain fractional-order chaotic systems and to incorporate an online function approximator (such as adaptive fuzzy system) to deal with model uncertainties.

Based on the universal approximation feature of the fuzzy systems [18], some adaptive fuzzy control schemes [19–25] have been developed for a class of uncertain chaotic systems with integer-order. In these schemes, the fuzzy systems are used to online approximate the uncertain nonlinear functions. The stability of the underlying closed-loop system has been analyzed in Lyapunov sense. In order to handle the bounded external disturbances as well as inevitable fuzzy approximation errors, a robust control term is added to the dominate fuzzy adaptive control term. This robust control term can be conceived by a sliding mode control approach [19–22] and an H_∞ control approach [23–25]. However, it is should be noted that these synchronization schemes [19–25] are limited to uncertain chaotic integer-order systems.

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http://dx.doi.org/10.1016/j.neucom.2015.08.003
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The chaos synchronization of fractional-order chaotic (or hyperchaotic) systems is yet considered as a challenging research topic [26–33]. In [26], chaos synchronization of variable-order fractional financial system based on active control method has been studied. In [27], a local stability criterion for synchronization of incommensurate fractional-order chaotic systems has been derived based on the stability theory of linear incommensurate fractional-order differential equations. A modified projective synchronization via active sliding mode control of two different fractional-order systems has been proposed in [28]. The author of [29] has designed an impulsive synchronization system for fractional order chaotic systems. Chaos synchronization between two different uncertain fractional order chaotic systems has been studied based on adaptive fuzzy sliding mode control in [30]. A generalized projective synchronization of fractional order chaotic system has been developed in [31] using an adaptive fuzzy sliding mode control strategy. In [32], an adaptive fuzzy control based synchronization of uncertain fractional-order chaotic systems with time delay has been proposed. In [33]. An adaptive fuzzy logic controller has been designed for achieving an H_∞ synchronizing for a class of uncertain fractional-order chaotic systems. However, the fundamental results of [30,32,33] are already questionable, because the stability analysis has not been derived rigorously in mathematics, as stated in [34,35].

In the current study, a direct adaptive fuzzy controller is designed to appropriately achieve a generalized projective synchronization of two different incommensurate fractional-order chaotic systems in which both uncertain dynamics and external disturbances are present. By using some coordinate transformation, non-fractional-order dynamics for the synchronization error are obtained. Thus, the stability analysis and controller design will be further simplified. A Lyapunov approach is adopted to carry out the design of the adaptation laws and the stability analysis of the corresponding closed-loop system. To show the effectiveness of the proposed synchronization system, some illustrative examples will be presented. Compared to the existing works [26–33], the principal contributions of this study can be summarized as:

1) The master system and the slave system, considered here, are assumed to be with non-identical structure, external dynamical disturbances, uncertain models and distinct fractional-orders. To the best of authors knowledge, the design of a direct adaptive fuzzy control for incommensurate fractional-order chaotic systems with all these mentioned properties has not been previously considered in the literature.
2) The conditions imposed in the previous literature [26–29] on full or partial knowledge of the models of the master and slave systems are neglected here. In fact, the adaptive fuzzy systems incorporated in the proposed controller permits to online estimate the uncertain functions.
3) Unlike the closely related works [30,32,33], the stability analysis of the underlying closed-loop system is strictly established in this paper, through the use of some properties of the Caputo fractional-order derivative [36–41].
4) The proposed controller can be applied without any difficulties to synchronize a large class of uncertain chaotic systems: e.g. to synchronize two identical incommensurate fractional-order chaotic systems, two distinct incommensurate fractional-order chaotic systems, two distinct commensurate fractional-order chaotic systems, two distinct integer-order chaotic systems, so on.
5) Compared to [31], our proposed controller is very simple, continuous and free of singularity problem (intrinsically related to indirect adaptive version). Moreover, one does not use the fractional-order derivatives of the master state vector as input for the designed fuzzy systems.

2. Basic definitions and preliminaries for fractional-order systems

The most frequently used definitions for fractional derivatives are: Riemann–Liouville, Grünwald–Letnikov, and Caputo definitions [36]. As the Caputo fractional operator is more consistent than another ones [36–41], then this operator will be employed in the rest of this paper. Also, a modification of Adams–Bashforth–Moulton algorithm proposed in [42,43] will be used for computer numerical simulation of the Caputo fractional-order differential equations.

The Caputo fractional derivative of a function $x(t)$ with respect to time is defined as follows [36]:

$$D_t^{\alpha} x(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{-\alpha+m-1} x^{(m)}(\tau) d\tau, \tag{1}$$

where $m=[\alpha]+1$, $[\alpha]$ is the integer part of α, D_t^{α} is called the $\alpha-$order Caputo differential operator, and $\Gamma(.)$ is the well-known Euler's gamma function:

$$\Gamma(P) = \int_0^{\infty} t^{P-1} e^{-t} dt; \text{ with } \Gamma(P+1) = P\Gamma(P) \tag{2}$$

This function can be seen as a natural extension of the factorial to real number arguments.

The following properties of the Caputo fractional-order derivative will be used in the sequent sections [36–38]:

Property 1. Let $0<q<1$, then

$$Dx(t) = D_t^{1-q} D_t^q x(t), \text{ where } D = \frac{d}{dt}. \tag{3}$$

Property 2. The Caputo fractional derivative operator is a linear operator:

$$D_t^q(\nu x(t) + \mu y(t)) = \nu D_t^q x(t) + \mu D_t^q y(t), \tag{4}$$

where ν and μ are real constants.

Especially, $D_t^q x(t) = D_t^q(x(t)+0) = D_t^q x(t) + D_t^q 0$, then, we have $D_t^q 0 = 0$.

Property 3. Consider a Caputo fractional nonlinear system [39–41]:

$$D_t^q x(t) = f(x(t)), \text{ with } 0<q<1 \tag{5}$$

If one assumes that $f(x(t))$ satisfies the Lipschiz condition with respect to x, i.e.,

$$\|f(x(t)) - f(x_1(t))\| \le \ell \|x(t) - x_1(t)\|, \tag{6}$$

where ℓ is a positive constant. Without loss of generality, one also assumes that $f(x)$ satisfies $f(x)=0$ at $x=0$.

$$\text{It follows that : } \|f(x(t))\| \le \ell \|x(t)\|. \tag{7}$$

Remark 1. The major advantage of the Caputo definition is that the initial conditions for fractional-order differential equations take on a similar form as for integer-order differential equations. In the literature, the Caputo definition is sometimes referred as a smooth fractional derivative.

3. Problem statement and fuzzy logic systems

3.1. Problem statement

Our main motivation consists in designing a fuzzy adaptive control system appropriately achieving a generalized projective synchronization between two different fractional-order chaotic systems. Fig. 1 presents the proposed synchronization scheme.

Please cite this article as: A. Boulkroune, et al., Fuzzy generalized projective synchronization of incommensurate fractional-order chaotic systems, Neurocomputing (2015), http://dx.doi.org/10.1016/j.neucom.2015.08.003