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Almost periodic solution for a neutral-type neural networks with distributed leakage delays on time scales

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ABSTRACT

By using exponential dichotomy theory, contraction mapping principle and discrete-continuous analysis method, we obtain some new sufficient conditions ensuring the existence and global exponential stability of the almost periodic solutions for a class of neutral-type neural networks with distributed leakage delays on time scales. An example is provided to demonstrate the usefulness of the main results obtained.

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1. Introduction

In the past few years, there have been a large number of researches on neutral-type neural networks due to its potential applications in many fields such as pattern recognition and automatic control, image and signal processing. Among various dynamical behaviors, the existence and stability of equilibrium points have proven to be the most important one that has received considerable research attention. For instance, if a neural network is employed to solve some optimization problems, it is highly desirable for the neural network to have a unique globally stable equilibrium, and it is not surprising that the stability analysis of neural networks has been an ever hot research topic resulting in enormous stability conditions reported in the literature, see e.g. [1,2,34,35].

Recently, a type of time-delay called leakage (or forgetting) delay has great impact on the community of neural networks, see e.g. [16–20]. A typical time delay called leakage (or forgetting) delay has great impact on the dynamical behavior of neural networks. In particular, it is worth pointing out that, in a nervous system, a typical time delay in the negative feedback terms which is known as leakage delays has a tendency to destabilize the system (see Gopalsamy [20]). Since leakage terms may have a destabilizing influence on the dynamical behaviors of neural networks, it is necessary to investigate leakage delay effects on the stability of neural networks (see [21]). For example, Park et al. [22] have investigated the synchronization problem for coupled neural networks with interval time-varying delays and leakage delay. In [20], the authors have studied the stability of equilibrium for the bidirectional associative memory (BAM) neural networks with constant delay in the leakage term. Balasubramaniam et al. [6] have studied the stability analysis for Takagi–Sugeno fuzzy cellular neural networks with mixed time-varying delays and time delay in the leakage term via the delay decomposition approach. More recently, Liu [16] has obtained the existence and stability results for general bidirectional associative memory neural networks with time-varying delays in the leakage terms by using fixed point theorem and Lyapunov functional.

It should be mentioned that most of the studies of the stability of real-valued neural networks are on continuous-time and discrete-time. In recent years, the theory of time scales has received a lot of attention, which was introduced by Stefan Hilger in his Ph.D. thesis [3] in 1988 in order to unify continuous and discrete analysis. Some authors were devoted to investigating the neural networks on time scales. For example, Cheng and Cao [4] have studied a complex networks with time delays on time scales by using Lyapunov functional and linear matrix inequality technique, and obtained several sufficient criteria to ensure the global exponential synchronization for the considered networks. Chen and Song [5] have considered the stability problem for the complex-valued neural networks with both leakage time delay and discrete time delay as well as two types of activation functions on time scales. In [7], a class of Cohen–Grossberg neural

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networks with distributed delays on time scales has been considered. Without assuming bounded conditions on these activation functions, the authors have established some sufficient conditions on the existence and global exponential stability of almost periodic solutions for Cohen–Grossberg neural networks on time scales. For more results on this subject, we refer the readers to [8–15] and references cited therein.

So far, to the best of the authors' knowledge, there are few results for the stability analysis to neutral-type neutral networks with distributed leakage delays on time scales. The major challenges are as follows: (1) in order to obtain existence and stability results, it is necessary for the neutral operator A to exist inverse operator on time scales. But how can we obtain its inverse operator A^{-1} and some properties about A^{-1}? (2) since almost periodic solution has special properties, the corresponding stability analysis becomes more complicated since a new mathematical analysis technique is required to reflect this influence; and (3) it is non-trivial to establish a unified framework to handle the influence of time scales, neutral terms and leakage delays. It is, therefore, the main purpose of this paper to make the first attempt to handle the challenges listed above.

In this paper, we consider the existence and stability problem for a neutral-type neural networks with leakage delays on time scales. Note that neural system with distributed leakage delays is dependent on the properties of neutral operator. We first develop a special analysis technique to account for the leakage delays and neutral time-delays. Then, exponential dichotomy theory and contraction mapping principle are utilized to derive sufficient conditions guaranteeing existence and global exponential stability of the almost periodic solutions. Finally, an example is presented to illustrate the usefulness and effectiveness of the main results obtained.

The following sections are organized as follows: In Section 2, some useful Lemmas and definitions are introduced. In Section 3, problem formulation is given. In Section 4, sufficient conditions are established for the existence of almost periodic solution of (3.1). The global exponential stability of (3.1) is studied in Section 5. In Section 6, an example is given to show the feasibility of our results. Finally, conclusions are drawn in Section 7.

2. Preliminaries

In this section, let us recall some definitions and lemmas about time scales, which are of importance in proving the main results of this paper.

Let $\mathbb{T}$ be a time scale which is a closed subset of $\mathbb{R}$. For $t \in \mathbb{T}$, the forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$, respectively, defined by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\}, \rho(t) = \sup\{s \in \mathbb{T} : s < t\}.$$

The point $t \in \mathbb{T}$ is called left-dense if $t > \inf\mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup\mathbb{T}$ and $\sigma(t) = t$, right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a right-scattered minimum m, set $\mathbb{T}_k := \mathbb{T} < \{m\}$, otherwise $\mathbb{T}_k := \mathbb{T}$. The backward graininess $\mu : \mathbb{T} \to [0, \infty)$ is defined by $\mu(t) = t - \rho(t)$. A function is called left-dense continuous provided it is continuous at left-dense point in $\mathbb{T}$ and its right-side limit exists at right-dense point in $\mathbb{T}$.

Definition 2.1 (*Bohner and Peterson [27]*). Let $f : \mathbb{T} \to \mathbb{R}$ be a function and $t \in \mathbb{T}_k$. Then define $f^{\nabla}(t)$ to be the number (provided it exists) with the property that given any $\varepsilon > 0$, there exits a neighborhood U of t such that

$$|f(\rho(t)) - f(s) - f^{\nabla}(\rho(t) - s)| \le \varepsilon |\rho(t) - s|,$$

for all $s \in U.f^{\nabla}$ is called to be the nabla derivative of f at t.

Let f, g be nabla differentiable functions on $\mathbb{T}$. Then

(i) $(c_1 f + c_2 g)^{\nabla} = c_1 f^{\nabla} + c_2 g$, for any constants c_1, c_2;
(ii) $(fg)^{\nabla} = f^{\nabla}(t)g(t) + f(\rho(t))g^{\nabla}(t) = f(t)g^{\nabla}(t) + f^{\nabla}(t)g(\rho(t))$;
(iii) If f and f^{∇} are continuous, then $(\int_a^t f(t,s)\nabla s)^{\nabla} = f(\rho(t), t) + \int_a^t f(t,s)\nabla s$;
(iv) If f is ld-continuous, then there exists a function F such that $F^{\nabla}(t) = f(t)$, and we define

$$\int_a^b f(t)\nabla(t) = F(b) - F(a).$$

Definition 2.2 (*Bohner and Peterson [27]*). A function $p : \mathbb{T} \to \mathbb{R}$ is said to be regressive provided $1 + \mu(t)p(t) \neq 0$ for all $t \in \mathbb{T}_k$, where $\mu(t) = \sigma(t) - t$ is the graininess function. The set of all regressive rd-continuous functions $f : \mathbb{T} \to \mathbb{R}$ is denoted by $\mathcal{R}_\mu$ while the set $\mathcal{R}_\mu^+$ is given by $\{f \in \mathcal{R}_\mu : 1 + \mu(t)f(t) > 0 \text{ for all } t \in \mathbb{T}\}$. Let $p \in \mathcal{R}_\mu$. The exponential function is defined by

$$\hat{e}_p(t,s) = \exp\left(\int_s^t \hat{\xi}_{\mu(\tau)}(p(\tau))\Delta\tau\right),$$

where $\hat{\xi}_h(z)$ is the so-called cylinder transformation.

Definition 2.3 (*Bohner and Peterson [27]*). If $p, q \in \mathcal{R}_\mu$, then a circle plus addition is defined by $(p \oplus_\mu q)(t) : = p(t) + q(t) - p(t)q(t)\mu(t)$, for all $t \in \mathbb{T}_k$. For $p \in \mathcal{R}_\mu$, we defined a circle minus p by $\ominus_\mu := -p/(1 - \mu p)$.

Lemma 2.1 (*Bohner and Peterson [27]*). *Let $p, q \in \mathcal{R}_\mu$. Then*

[i] $\hat{e}_0(t,s) \equiv 1$ *and* $\hat{e}_p(t,t) \equiv 1$;
[ii] $\hat{e}_p(\rho(t), s) = (1 - \mu(t)p(t))\hat{e}_p(t,s)$;
[iii] $\hat{e}_p(t,s) = \frac{1}{\hat{e}_p(s,t)}$;
[iv] $\hat{e}_p(t,s)\hat{e}_p(s,r) = \hat{e}_p(t,r)$;
[v] $\hat{e}_p^{\nabla}(\cdot, s) = p\hat{e}_p(\cdot, s)$;

Definition 2.4 (*Bohner and Peterson [27]*). A time scale $\mathbb{T}$ is called an almost periodic time scale if

$$\Pi := \{\tau \in \mathbb{R} : t \pm \tau \in \mathbb{T}, \forall t \in \mathbb{T}\} \neq \{0\}.$$

Definition 2.5 (*Li and Wang [28]*). Let $\mathbb{T}$ be an almost periodic time scale. A function $f \in C(\mathbb{T}, \mathbb{R})$ is called an almost periodic function if the ε-translation set of f

$$E\{\varepsilon, f\} = \{\tau \in \Pi : |f(t+\tau) - f(t)| < \varepsilon, \forall t \in \mathbb{T}\}$$

is a relatively dense set in $\mathbb{T}$ for all $\varepsilon > 0$; that is, for any given $\varepsilon > 0$, there exists a constant $l(\varepsilon) > 0$ such that each interval of length $l(\varepsilon)$ contains a $\tau(\varepsilon) \in E \in \{\varepsilon, f\}$ such that

$$|f(t+\tau) - f(t)| < \varepsilon, \forall t \in \mathbb{T},$$

where τ is called the ε-translation number of and $l(\varepsilon)$ is called the inclusion length of $E\{\varepsilon, f\}$.

Definition 2.6 (*Li and Wang [29]*). Let $A(t)$ be an $n \times n$ matrix valued function on $\mathbb{T}$. Then the linear system

$$x^{\nabla}(t) = A(t)x(t), \ t \in \mathbb{T}, \tag{2.1}$$

is said to admit an exponential dichotomy on $\mathbb{T}$ if there exist constants $k_i, \alpha_i, i = 1, 2$, projection P and the fundamental solution matrix $X(t)$ of (2.1) satisfying

$$\|X(t)PX^{-1}(s)\| \le k_1 \hat{e}_{\ominus_\mu \alpha_1}(t,s), \ s, t \in \mathbb{T}, t \ge s,$$

$$\|X(t)PX^{-1}(s)\| \le k_2 \hat{e}_{\ominus_\mu \alpha_2}(t,s), \ s, t \in \mathbb{T}, t \ge s,$$

Lemma 2.2 (*Li and Wang [30], Wang [31]*). *If the linear system (2.1) admits an exponential dichotomy, then the almost periodic system*

$$x^{\nabla}(t) = A(t)x(t) + g(t), \ t \in \mathbb{T},$$

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