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Global attracting sets of non-autonomous and complex-valued neural networks with time-varying delays

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ABSTRACT

In this paper, a class of non-autonomous and complex-valued neural networks with time-varying delays are investigated. Some sufficient conditions to guarantee the boundedness of the networks are derived, by developing a new integral inequality and applying the properties of spectral radius of nonnegative matrix. Meanwhile, the framework of the global attracting sets for complex-valued neural networks with time-varying delays is given out. Finally, the effectiveness of the theoretical analysis is illustrated by some examples with computer simulation.

1. Introduction

In recent years, the study of the complex-valued neural networks (CVNNs for short) is the growing field of research, due to extensive applications in many engineering areas [1–5], such as information, electrical engineering, and control engineering. In [3], an optimal control scheme was proposed based on adaptive dynamic programming (ADP) algorithm for complex-valued systems. In [4], a channel prediction method was proposed based on the CVNNs, which can bring about both high flexibility and low generalization error in channel predictions. In [5], CVNNs were utilized to realize the group-movement of mobile robots with multiple sensors. For more applications, readers may refer to [10–15,23,29] and the references therein.

The above-mentioned applications depend largely on the dynamic behavior of the CVNNs. Therefore, it is important to investigate the dynamic behavior of the CVNNs. Recently, there are various types of conditions to guarantee asymptotical stability, exponential stability and complete stability or multistability for complex-valued neural networks, which are more different and complicated than real-valued ones. In [6–8], some sufficient conditions to ensure the existence, uniqueness and globally asymptotical stability of the equilibrium point for CVNNs with time-delays were obtained. In [6,9,10], the authors have studied global activation dynamics of CVNNs and obtained some new criteria for global exponential stability of the unique equilibrium pattern. The monostable neural networks guarantee that all the trajectories converge to one equilibrium point. However, sometimes the networks may have multi-equilibrium points. Thus, the study of multistability analysis of CVNNs has received increasing interest. Boundedness, global attractivity and complete stability are three basic properties of multistable neural networks that make the each trajectory of the CVNNs converges to the set of the equilibrium points. In [11–14,24,27,28,30], some of the existing results for multistability analysis are obtained. To the best of our knowledge, all the CVNNs concerning the above-mentioned dynamic behavior are autonomous systems.

In many real physical systems, the equilibrium points sometimes do not exist, especially in nonlinear and non-autonomous neural networks with time delays. Therefore, it motivated many researchers to discuss the attracting sets of neural networks with time delays. It is known that inequality technique is one of the critical methods to investigate the attracting sets [14–21,25,26,31]. The inequality in [14,20] is ineffective for studying the global attracting sets of non-autonomous and complex-valued neural networks with delays. In [21], Xu derived a new integral inequality to investigate the dynamic behavior of the integro-differential equation. In [16,17], Xu et al. studied the attracting sets of non-autonomous neural networks with delays, by using the integral inequality in the above-mentioned works.

Motivated by the above discussions, this paper considers the global attracting sets of a class of non-autonomous and complex-valued networks with time-varying delays, by establishing new integral inequalities. Some new sufficient conditions for the existence of the boundedness and global attracting sets are obtained. The remainder of this paper is organized as follows. In Section 2, some

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preliminaries including some necessary definitions, hypotheses are described. The main results are presented in Section 3. In Section 4, two numerical examples are given to demonstrate the theoretical results. Finally, the conclusions of this paper are given in Section 5.

Notations: Throughout this paper, the following notations are used. Let the subscript T denote the matrix transposition. E denotes unit matrix, R is the set of real numbers and $R_+ = [0, +\infty)$, R^n denotes the n-dimensional Euclidean space. $\mathbb{C}$ is the set of complex numbers and i represents the imaginary unit. $z(t)$ presents the complex-valued function, $z(t) = x(t) + iy(t)$ where $x(t), y(t) \in R^n$. For any matrix $A, B \in R^{n\times n}$, $A \le B$ means that each pair of corresponding elements of A and B satisfies the inequality "$\le$". Especially, A is called a nonnegative matrix if $A \ge 0$. For square matrix A, A^{-1} denotes its inverse, and $\rho(A)$ denotes its spectral radius. $C(X,Y)$ denotes the space of continuous mapping from the topological space X to the topological space Y. Especially, let $C \triangleq C((-\infty, t_0], R^n)$ with $\phi \in C$ is bounded on $(-\infty, t_0]$.

For $x(t) \in C(R, R^n)$, $\tau(t) \in C(R, R_+)$, we define $\|x(t)\| = (|x_1(t)|, |x_2(t)|, \ldots, |x_n(t)|)$, $|x_i(t)| \ge 0$; $\|x(t)\|_{\tau(t)} = (\|x_1(t)\|_{\tau(t)}, \|x_2(t)\|_{\tau(t)}, \ldots, \|x_n(t)\|_{\tau(t)})$, $\|x_i(t)\|_{\tau(t)} = \sup_{0\le s\le \tau(t)} |x_i(t-s)|$, $i = 1, 2, \ldots, n$.

2. Preliminaries

In this paper, we consider the non-autonomous CVNNs with time-varying delays:

$$\dot{Z}(t) = -A(t)Z(t) + B(t)f(Z(t)) + D(t)f(Z(t-\tau(t))) + H(t), \tag{1}$$

where $Z(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^T \in \mathbb{C}^n$ is the state vector of the neural network with n neurons. $A(t) = \text{diag}\{a_1(t), a_2(t), \ldots, a_n(t)\} \in R^{n\times n}$ with $a_i(t) > 0$ is the self-feedback connection weight matrix. $B(t) = (b_{ij}(t))_{n\times n} \in \mathbb{C}^{n\times n}$ is the connection weight matrix, $D(t) = (d_{ij}(t))_{n\times n} \in \mathbb{C}^{n\times n}$ is the delay connection weight matrix. $H(t) = (h_1(t), h_2(t), \ldots, h_n(t))^T \in \mathbb{C}^n$ is the external input vector. $\tau(t) \in C(R, R_+)$ is the transmission delays, $\lim_{t\to+\infty}(t-\tau(t)) = +\infty$. $f(\cdot)$ is the activation function defined by

$$f(z) = \max(0, \text{Re}(z)) + i\max(0, \text{Im}(z)),$$

and $f(Z(t)) = (f(z_1(t)), f(z_2(t)), \ldots, f(z_n(t)))^T \in \mathbb{C}^n$. The initial conditions associated with the neural network (1) are given by

$$z_i(s) = \phi_i(s), \quad -\infty \le s \le t_0, \quad i = 1, 2, \ldots, n,$$

where $\text{Re}(\phi_i(s))$ and $\text{Im}(\phi_i(s))$ are continuous on $(-\infty, t_0]$. Let $x_i(t) = \text{Re}(z_i(t)), y_i(t) = \text{Im}(z_i(t))$, $b_{1ij}(t) = \text{Re}(b_{ij}(t)), b_{2ij}(t) = \text{Im}(b_{ij}(t)), d_{1ij}(t) = \text{Re}(d_{ij}(t)), d_{2ij}(t) = \text{Im}(d_{ij}(t)), h_{1i}(t) = \text{Re}(h_i(t)), h_{2i}(t) = \text{Im}(h_i(t)), \phi_{1i}(t) = \text{Re}(\phi_i(t)), \phi_{2i}(t) = \text{Im}(\phi_i(t))$.

Now the non-autonomous CVNNs (1) can be rewritten as

$$\begin{aligned}
\dot{x}_i(t) &= -a_i(t)x_i(t) + \sum_{j=1}^n b_{1ij}(t)f(x_j(t)) - \sum_{j=1}^n b_{2ij}(t)f(y_j(t)) \\
&\quad + \sum_{j=1}^n d_{1ij}(t)f(x_j(t-\tau(t))) - \sum_{j=1}^n d_{2ij}(t)f(y_j(t-\tau(t))) + h_{1i}(t), \\
\dot{y}_i(t) &= -a_i(t)y_i(t) + \sum_{j=1}^n b_{2ij}(t)f(x_j(t)) + \sum_{j=1}^n b_{1ij}(t)f(y_j(t)) \\
&\quad + \sum_{j=1}^n d_{2ij}(t)f(x_j(t-\tau(t))) + \sum_{j=1}^n d_{1ij}(t)f(y_j(t-\tau(t))) + h_{2i}(t),
\end{aligned} \tag{2}$$

for all $i = 1, 2, \ldots, n$. The initial conditions of non-autonomous CVNNs (2) are given in the following form:

$$\begin{cases} x_i(s) = \phi_{1i}(s) \\ y_i(s) = \phi_{2i}(s) \end{cases}, \quad s \in (-\infty, t_0], \quad i = 1, 2, \ldots, n.$$

In the following, we will introduce some definitions and lemmas that are necessary.

Definition 1. The non-autonomous CVNNs (1) is said to be bounded if the real and imaginary parts of its each trajectory are bounded.

Definition 2. If there exists a compact set $S \subset \mathbb{C}^n$ such that for any initial value $\phi^T \in \mathbb{C}^n \backslash S$, $\lim\sup_{t\to+\infty} d(Z^T(t), S) = 0$, then S is said to be a globally attracting set of (1), where $\mathbb{C}^n \backslash S$ is the complement set of S, $d(Z^T(t), S)$ denotes the distance of $Z^T(t)$ to S in $\mathbb{C}^n$.

This means, if each trajectory $Z(t) = (z_1(t), z_2(t), \ldots, z_n(t))^T$ of neural network (1) satisfies

$$\begin{cases} \lim\limits_{t\to+\infty} \sup |\text{Re}(z_p(t))| \le s_p^R < +\infty, \\ \lim\limits_{t\to+\infty} \sup |\text{Im}(z_p(t))| \le s_p^I < +\infty, \end{cases}$$

for $p = 1, 2, \ldots, n$, then set

$$\Omega = \{x_p(t) + iy_p(t) \in \mathbb{C} : |x_p(t)| \le s_p^R, |y_p(t)| \le s_p^I, \quad p = 1, 2, \ldots, n\}.$$

is a globally attracting set of neural network (1).

Definition 3 (Xu [21]). $f(t,s) \in UC_t$ means that $f \in C(R_+ \times R, R_+)$ and for any given α and any $\varepsilon > 0$ there exist positive numbers μ, T and ν satisfying

$$\int_\alpha^t f(t,s)\, ds \le \mu, \quad \int_\alpha^{t-T} f(t,s)\, ds \le \varepsilon, \quad \forall t \ge \nu.$$

Especially, $f \in UC_t$ if $f(t,s) = f(t-s)$ and $\int_0^{+\infty} f(u)\, du < \infty$.

Lemma 1 (*Berman and Plemmons [22]*). *If $A \ge 0$ and $\rho(A) < 1$, then*

(a) $(E-A)^{-1} \ge 0$
(b) *there is a positive vector $z \in \Omega_\rho(A)$ such that $(E-A)z > 0$.*

For convenience, we denote $B_1(t) = (|b_{1ij}(t)|)_{n\times n}, B_2(t) = (|b_{2ij}(t)|)_{n\times n}$, $D_1(t) = (|d_{1ij}(t)|)_{n\times n}, D_2(t) = (|d_{2ij}(t)|)_{n\times n}, H_1(t) = (|h_{11}(t)|, |h_{12}(t)|, \ldots, |h_{1n}(t)|)^T, H_2(t) = (|h_{21}(t)|, |h_{22}(t)|, \ldots, |h_{2n}(t)|)^T$. The following hypotheses are hold to derive the main results.

(A1) $\forall t \ge t_0$, there exist nonnegative constant matrices $\gamma, \overline{\gamma}$ and constant vectors $\kappa_1 \ge 0, \kappa_2 \ge 0$ such that

$$\begin{aligned}
&\int_{t_0}^t e^{-\int_s^t A(v)\, dv}(B_1(s) + D_1(s))\, ds \le \gamma, \\
&\int_{t_0}^t e^{-\int_s^t A(v)\, dv} H_1(s)\, ds \le \kappa_1; \\
&\int_{t_0}^t e^{-\int_s^t A(v)\, dv}(B_2(s) + D_2(s))\, ds \le \overline{\gamma}, \\
&\int_{t_0}^t e^{-\int_s^t A(v)\, dv} H_2(s)\, ds \le \kappa_2.
\end{aligned} \tag{3}$$

(A2) $\rho(\gamma) < 1, \gamma + \overline{\gamma} \ge E_n$.
(A3) $\omega_i \triangleq \inf_{t_0\le s\le t} \int_s^{s+\theta} a_i(v)\, dv > 0$ for some $\theta > 0, i = 1, 2, \ldots, n$.

3. Main results

In this section, we firstly establish the integral inequality with delays, then obtain the sufficient conditions to guarantee the boundedness of the non-autonomous CVNNs (1) by applying the integral inequality. Finally, the global attracting sets of the network are given out.

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