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Fuzzy associative memories: A design through fuzzy clustering

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ABSTRACT

In this study, we discuss a design of fuzzy associative structures (memories) realized within the framework of fuzzy clustering. Associative memories are inherently direction-free structures (and the recall of objects can be realized for any variable or a subset of variables). Fuzzy clustering being direction-free comes here as a sound design alternative. Two recall proposals are studied: one involves prototypes (being the key descriptors of the structure of the data) and their activation in the presence of partially available data to be recalled whereas the second proposal involves fuzzy correlation matrices and in principle exhibits some resemblance with a standard correlation associative memories. In the setting of associative memories, Fuzzy C-Means (FCM) is studied. The recall error is discussed with regard to the essential parameters of the FCM (the number of clusters and the fuzzification coefficient). Furthermore we discuss an optimization of the distance function used in the clustering algorithm realized with regard to the recall error. Experimental results are provided along with a comparative study involving correlation-based associative memories.

1. Introduction

Associative memories are architectures aimed at storing and recalling associations among patterns (data) [10,11,18]. The essence of recall is that in associative memories associative recall (retrieval) produces a pattern associated with input data. In contrast to random access memories (where an item is obtained through its address), associative memories are content-addressable. Formally speaking, given a collection of associations-pairs of linked (associated) data ($\boldsymbol{x}_k$, $\boldsymbol{y}_k$), k=1, 2, …,$\boldsymbol{N}$ (with $\boldsymbol{x}_k$ and $\boldsymbol{y}_k$ being vectors in n- and p-dimensional Euclidean space, respectively), these pairs are stored in an associative fashion in a certain memory (mapping). Let us denote this mapping by M. Next, an associative recall retrieves the missing item in the pair. Associative recall is direction-free, meaning that the memory should be able to recall $\boldsymbol{x}_k$ given $\boldsymbol{y}_k$ and vice-versa: $\boldsymbol{y}_k$ is recalled in the presence of $\boldsymbol{x}_k$. Formally speaking, $\boldsymbol{y}_k=M(\boldsymbol{x}_k)$ and $\boldsymbol{x}_k=M(\boldsymbol{y}_k)$ (let us stress that M is direction free (say, a correlation matrix) and thus the mechanisms of recall are realized in any direction.

There are two main categories of associative memories. An auto-associative memory can deal with problems when the input patterns and output patterns are the same ($\boldsymbol{x}_k=\boldsymbol{y}_k$). When the input and output patterns are different, we are concerned with a hetero-associative memories. From the design perspective, there are a number of alternatives starting from the classic correlation associative memory proposed by Kohonen [10] in which the memory is produced in the form of the correlation matrix. The key challenges are about increasing a capacity of the memory so that a large number of patterns can be stored and effectively recalled. Achieving high robustness of the recall mechanism is another design challenge.

Ideally, the recall mechanism should not produce any recall error meaning that if the input is equal to $\boldsymbol{x}_k$, then the recalled output should be equal to $\boldsymbol{y}_k$. It is also anticipated that associative memories exhibit some robustness meaning that if $\boldsymbol{x}$ is very similar to $\boldsymbol{x}_l$ then the recall returns $\boldsymbol{y}$ that is either equal to $\boldsymbol{y}_l$ or is positioned very close to it. There have been a number of alternative architectures reported in the literature [6,8,13,15,16,17,23,24,27,28,36,39,40,42,43]. Furthermore from application perspective [9,19,25,32,33,37,41], associative memories support various modeling scenarios. For instance input–output data ($\boldsymbol{x}_k$, $\boldsymbol{y}_k$) used in system modeling (multiple input-single output system) give rise to a memory, which can be regarded as the underlying model of the system. The recall of the output $\boldsymbol{y}$ on a basis of the input $\boldsymbol{x}$ can be sought as prediction while the recall of $\boldsymbol{x}$ for the output $\boldsymbol{y}$ given can be sought as an inverse problem in system modeling. One can think of a recall process realized for spatio-

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temporal data. Consider a spatiotemporal data where we have a number of weather stations distributed over a certain region. Each station positioned at some ($\boldsymbol{x}$, $\boldsymbol{y}$) coordinates collects a series of measurements ($\boldsymbol{z}_1$, $\boldsymbol{z}_2$, …,$\boldsymbol{z}_m$). There are a number of practically oriented recall scenarios: (a) the position ($\boldsymbol{x}$,$\boldsymbol{y}$) is given and several measurements are given. Determine the values of the missing readings, (b) the values of measurements are given; we are interested in determining possible locations where these measurements could come from.

In the setting of fuzzy sets, the concept of fuzzy associative memories [12] has been around for a long time. Here the items are vectors in [0, 1] hypercube and the recall mechanism is driven by the sup-min or sup-t composition operator, which operates on a given input and a fuzzy relation build as a union of the Cartesian products of the individual pairs of items ($\boldsymbol{x}_k$, $\boldsymbol{y}_k$) (which can be regarded as a close analog of the correlation matrix used in correlation associative memories [10]). In terms of popularity of fuzzy associative memories of this type, one can note that sometimes fuzzy controllers (rule-based systems) are regarded as fuzzy associative memories with the input–output fuzzy sets present in each rules being treated as pairs of items stored in the memory. Such fuzzy associative memories suffer from a limited storage capacity. To overcome the limitation several improved fuzzy associative memories are derived. For example, Chung and Lee generalized Kosko's model by using a max-bounded product composition such that all fuzzy rules can be perfectly recalled by a single fuzzy associative memory matrix if the input patterns satisfy some orthogonality conditions [3]. Fan et al. [5] presented an improved learning rule for the generic fuzzy associative memories. Liu [14] further improved Fan's work by introducing a threshold mechanism. Bělohlávek [1] discussed a fuzzy logic bidirectional associative memory. Wang and Lu [38] developed fuzzy morphological associative memories which exhibit some level of robustness. Sussner and Valle [29] proposed implicative fuzzy associative memories. Some further works by Sussner et al. [30,31,34,35]. Goh et al. [7] concentrated on fuzzy associative conjuncted maps.

Having a look at the multi-directional aspect of functioning of associative memories, in this setting, the direction-free (relational) character of the memories calls for a suitable development setting. Here fuzzy clustering arises as a sound and legitimate alternative. Fuzzy clustering is direction-free: in virtue of the underlying optimization and a form of the objective function, there is no distinction between input and output variables. Fuzzy clusters reveal structure in the data. Prototypes come as representatives of the data and in the context of associative memories form the memory. The quality of recall is implied by the abilities of the fuzzy cluster to represent the data. The reconstruction error [21] is a useful measure to assess the quality of recall. In the study we capitalize on the relational nature of constructs of fuzzy clusters showing that such information granules (clusters) help us realize a multidirectional nature of recall (reconstruction). The objective is to develop fuzzy cluster-based associative memories. The two recall mechanisms are introduced. The first one exploits a collection of prototypes and engages a reconstruction process. The second one realizes a recall on a basis of the obtained fuzzy correlation matrices and in this way is closely positioned to the concept of the correlation associative memories. The direct motivation behind these cluster-based associative memories is twofold. First, clusters reflect a structure of the data and one may expect that forming an associative memory of this structure can enhance the quality of recall. Second, associative memories are direction-free and so are clustering techniques. As a matter of fact, this avenue of research, although quite promising and legitimate, has not been pursued in the past.

The exposure of the material is arranged into 6 sections. In Section 2, we formulate a concept of a fuzzy associative memory and discuss a recall problem. Several ways of optimization of the recall mechanism are identified and studied in Section 3. In the sequel, in Section 4, we discuss a fuzzy correlation memory in which we exploit the structural information (prototypes and partition matrix) supplied by fuzzy clustering. Experimental results are reported in Section 5; here we offer a thorough comparative analysis. Conclusions are drawn in Section 6.

In this study, we are concerned with auto-associative memories meaning that $\boldsymbol{x}_k$ is associated with itself and a recall is about determining missing coordinates of the given incomplete pattern.

2. Formulation of the fuzzy associative memory and a realization of the recall problem

Fuzzy clustering forms a convenient vehicle to realize associative memory. In this section, we focus on the buildup of the memory showing a nature of the associative memory and discuss a recall mechanism, which relates directly with the reconstruction capabilities of the clusters.

Consider a collection of N data (patterns) positioned in an n-dimensional space of real numbers $\boldsymbol{R}^n$, namely $\boldsymbol{x}_1$, $\boldsymbol{x}_2$, …,$\boldsymbol{x}_N$. We build c clusters-information granules which are fully described by their prototypes $\boldsymbol{v}_1$, $\boldsymbol{v}_2$, …,$\boldsymbol{v}_c$. Along with the prototypes formed is a partition matrix $\boldsymbol{U}=[u_{ik}]$, $i=1, 2,…,c$; $k=1,2, …,N$. This characterization of the structure of data through clusters (information granules) is typical for objective function-based clustering methods. Here Fuzzy C-Means (FCM) [2,20] is commonly encountered in the technology of fuzzy sets. The reconstruction criterion [21] studied in this setting is also beneficial in the quantification of the quality of recall. Fuzzy clustering serves as a conceptual and algorithmic vehicle to reveal structure in the data and effectively utilize it to carry out recall. In this way, FCM is completed by minimizing the standard objective function and the reconstruction criterion is a vehicle to enhance the quality of recall.

Clustering is a direction-free process. In the process of revealing a structure there is no distinction between input and output variables. This feature is essential to associative memories. The direction-free character of clustering means that the revealed structure in the data is evidently *relational* and because of this becomes a suitable framework to realize the mechanisms of associative recall. The associative memory realized by the FCM clustering comes as a family (set) of prototypes {$\boldsymbol{v}_1$, $\boldsymbol{v}_2$, …,$\boldsymbol{v}_c$}. The recall process is about retrieving (recalling) a missing (unknown) variable on a basis of the memory (a collection of the prototypes) and the available coordinates of the data.

The crux of the recall processing is formulated as follows:

1. For a vector of data (pattern) $\boldsymbol{x}$ with the coordinates in $\boldsymbol{R}^m$ where $m<n$, recall (reconstruct) the missing entries of the pattern.

If $m=n-1$, we are concerned with a single variable recall through invoking associations. If $m=n-k$ there is a k-variable recall involving k variables. Let us remind that the simplest version of the problem comes when a single variable is recalled ($m=n-1$). The diversity of the recall processes with regard to the variety of variables to be recalled is visualized in Fig. 1.

Proceeding with the relational structure formed by the fuzzy clusters, we look at the detailed calculations supporting the realization of the recall process. Let us remind that the FCM clustering is realized in the presence of all variables.

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