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A novel method to solve inverse variational inequality problems based on neural networks

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ABSTRACT

This paper presents a neural network for solving the inverse variational inequality problems. The proposed neural network possesses a simple one-layer structure and is suitable for parallel implementation. It is shown that the proposed neural networks are globally convergent to the optimal solution of the inverse variational inequality and are globally asymptotically stable, and globally exponentially stable, respectively under different conditions. Numerical examples are provided to illustrate the effectiveness and satisfactory performance of the neural networks.

1. Introduction

In this paper, we are concerned with solving a class of inverse variational inequalities denoted by $IVI(\Omega, F)$:find $u_* \in R^n$ such that $F(u_*) \in \Omega$, and

$$(\phi - F(u_*))^T u_* \geq 0, \quad \forall\, \phi \in \Omega \tag{1}$$

where Ω is a closed convex set of designated constraints. If an inverse function $u = F^{-1}(x) = f(x)$ exists, the above IVI problem could be transformed as a regular variational inequality denoted by $VI(\Omega, f)$:find $x_* \in \Omega$, such that

$$(x - x_*)^T f(x_*) \geq 0, \quad \forall\, x \in \Omega \tag{2}$$

However, in real life, it is common that the inverse function of $F^{-1}(x)$ does not have a explicit form. For instance, in network equilibrium problems, state variables x represent an equilibrium flow pattern under given controls u. It is almost impossible to find an explicit functional relationship between x and u [1].

He et al. [2] introduced the IVI problem and proposed a projection-based iterative algorithm for solving the IVI problems and applied it to solve a bipartite market equilibrium problem. IVI problems have numerous applications in science and engineering. As mentioned in [1], in addition to the market equilibrium problem in economics, a number of normative flow control problems appearing in transportation and telecommunication networks could be transformed into IVI problems. In many of the aforementioned applications of IVI, real-time solutions of problem (1) are always imperative. However, conventional iterative numerical methods may not be efficient since the computing time required for a solution is greatly dependent on the dimension and the structure of the problem.

In recent years, recurrent neural networks have found successful applications in signal processing, pattern recognition, associative memory and other engineering or scientific fields [3–11]. Moreover, owing to the inherent nature of parallelization and distributed information process, neural networks have served as promising computational models for realtime applications. So in recent decade, neural networks designed for solving mathematic programming and related optimization problems have been intensively studied in [12–17]. For instance, [12] proposed a projection neural network for solving a class of nonlinear variational inequality with box constrained. Hu and Wang [16] developed an improved dual neural network to solve the assignment problems in realtime. Xia et al. [17] proposed a discrete-time neural network for fast solving large linear L_1 estimation problems and applied it into image restoration. However, to the best of the authors' knowledge, there is no existing neural network which is employed to solve IVI or related problems.

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In this paper, we propose a recurrent neural network with a single-layer structure to deal with the IVI problem (1). The proposed network is proved to be globally convergent to the exact solution of the IVI (1). By the theory of functional differential equation and Lyapunov, the globally asymptotical and exponential stability criterion of the proposed neural network can be obtained under various conditions. Finally, the effectiveness of the proposed neural network is illustrated by numerical examples.

The rest of this paper is organized as follows. In the next section, a projection neural network model for solving the IVI problem (1) is presented. In Section 3, we study the global stability of the proposed neural network, including the global asymptotic stability, and global exponential stability. In Section 4, we present numerical examples to illustrate the effectiveness of the neural network for solving IVI problems. Section 5 concludes this paper.

2. Preliminaries

Based on the well-known projection theorem [18], it follows that $x^* \in \Omega$ is a solution of the $VI(\Omega, f)$ defined in (2) if and only if it satisfies the following projection equation:

$$x = P_\Omega[x - \alpha f(x)], \tag{3}$$

where $\alpha > 0$ is a constant and P_Ω: $R^n \to \Omega$ is a projection operator defined by $P_\Omega(x) = [P_\Omega(x_1), \ldots, P_\Omega(x_n)]^T$, and

$$P_\Omega(x_i) = \arg\min \; \| x - v \|, \quad v \in \Omega, \tag{4}$$

where $\|\cdot\|$ denotes l_2 norm.

Following a similar line, solving the $IVI(\Omega, F)$ (1) is equivalent to solve the projection equation as follows:

$$F(u) = P_\Omega[F(u) - \alpha u]. \tag{5}$$

Based on the above discussions, we propose the following neural network to solve the $IVI(\Omega, F)$ (1):

$$\frac{du(t)}{dt} = \lambda\{P_\Omega[F(u) - \alpha u] - F(u)\}, \tag{6}$$

where $\lambda > 0$ is a scaling factor and P_Ω is defined in (4). It is obvious that u^* is a solution of $IVI(\Omega, F)$ (1) if and only if it is the equilibrium point of the above neural network. The network (6) can be easily realized by a recurrent neural network with a one-layer structure as shown in Fig. 1. The projection operator $P_\Omega(\cdot)$ can be implemented by using a piece-wise activation function. According to Fig. 1, the circuit realizing the presented neural network (6) consists of n integrators, n piece-wise activation functions, n processors for $U(x)$ and $2n$ summators. Therefore, the proposed network complexity only depends on the function $F(x)$ in the original problem. Furthermore, the proposed network can be implemented in a circuit with simple hardware, unlike the numerical projection methods which cannot be implemented in the circuit due to the choice of the varying step length.

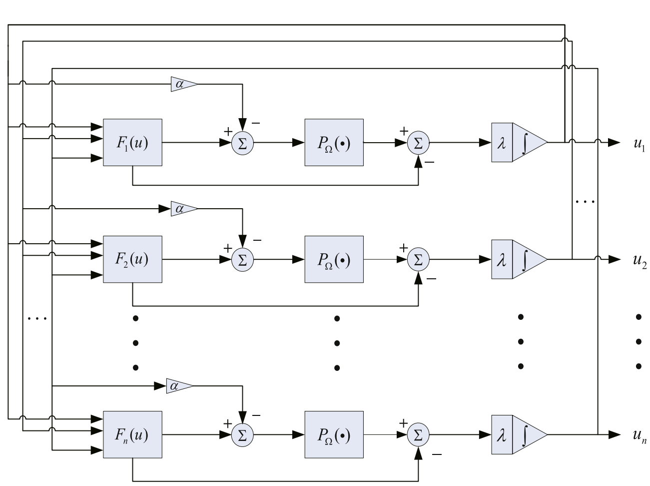

Fig. 1. Architecture of the proposed neural network (6).

For the convenience of later discussions, it is necessary to introduce a few definitions and lemmas.

Definition 1. A function F: $R^n \to \Omega \subset R^n$ is said to be Lipschitz continuous with constant L on the set Ω if, for every pair of points $x, y \in \Omega$,

$$\| F(x) - F(y)\| \le L \| x - y \| \tag{7}$$

F is said to be locally Lipschitz continuous on Ω if each point of Ω has a neighborhood $D \subset \Omega$ such that the above inequality holds for each pair of points $x, y \in \Omega$.

Definition 2. A mapping F: $R^n \to \Omega \subset R^n$ is said to be monotone on Ω if, for every pair of $x, y \in \Omega$,

$$\langle F(x) - F(y), x - y\rangle \ge 0; \tag{8}$$

where $\langle\cdot,\cdot\rangle$ denotes an inner product.

F is said to be strongly monotone on Ω if, for each pair of $x, y \in \Omega$, there exists a constant $\varrho > 0$ such that

$$\langle F(x) - F(y), x - y\rangle \ge \varrho \| x - y \|^2. \tag{9}$$

Definition 3. A dynamic system is said to be globally exponentially stable at x^* if every trajectory starting at any initial point $x(t_0) \in R^n$ satisfies

$$\| x(t) - x^* \| \le \gamma \cdot e^{-\xi(t-t_0)}, \quad \forall\, t \ge t_0 \tag{10}$$

where γ and ξ are positive constants independent of the initial points. It is clear that the exponential stability implies the globally asymptotical stability and the system converges arbitrarily fast.

Lemma 1. *Let the P_Ω be the projection operator defined in (4), for any $x \in R^n$, $y \in \Omega \subset R^n$:*

$$(x - P_\Omega(x))^T(P_\Omega(x) - y) \ge 0, \tag{11}$$

and for any $x, y \in R^n$:

$$\| P_\Omega(x) - P_\Omega(y)\| \le \| x - y \|. \tag{12}$$

Lemma 2 (*Gronwall inequality, Beesack [19]*). *Let $\hat{u}$ and $\hat{\omega}$ be real-valued nonnegative continuous functions with domain $(t|t \ge t_0)$; let $a(t) = a_0(|t - t_0|)$, where a_0 is a monotone increasing function; assume that, for $t \ge t_0$*

$$\hat{u}(t) \le a(t) + \int_{t_0}^{t} \hat{u}(s)\hat{\omega}(s)\, ds, \tag{13}$$

then

$$\hat{u}(t) \le a(t)\exp\int_{t_0}^{t} \hat{\omega}(s)\, ds.$$

Lemma 3. *For any $a \in R^n$, $b \in R^n$, the following inequality holds:*

$$a^T b \le \| a \| \| b \|. \tag{14}$$

3. The neural networks' global stability analysis

First, the existence and uniqueness of the solution of the proposed neural network defined by (6) is analyzed.