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# Exponential stability of stochastic neural networks with leakage delays and expectations in the coefficients ☆

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ABSTRACT

This paper introduces a new class of stochastic neural networks with time delays in the leakage terms. A key characteristic of this new stochastic neural network is that its coefficients are dependent on expectations. We first establish a novel stability lemma for this new model. Then, by applying this new stability lemma, Itô's formula, Lyapunov–Krasovskii functional, stochastic analysis theory and matrix inequalities technique, we show that the suggested system is exponentially stable in the mean square. Moreover, some remarks and discussions are given to illustrate that the obtained results are significant, which generalizes and comprises those obtained in the previous literature. Finally, an example is given to show the effectiveness of the theoretical result.



## 1. Introduction

As is well known, it was reported in [1–5] that neural networks can be applied to many important fields such as image processing, optimization problems, pattern recognition, signal processing, associative memory, fault diagnosis, and so on. As a consequence, much attention was paid to neural networks in the past decades. Indeed, various classes of neural networks such as Hopfield neural networks, Cohen–Grossberg neural networks, bidirectional associative memory neural networks and cellular neural networks have been widely studied in the literature. One of the best important topic in the field of neural networks is the stability research.

Besides, due to the finite speed of information processing, time delays are always unavoidably encountered in the implementation of neural networks. Moreover, the existence of time delays may lead to oscillation and instability of neural networks, which is harmful to the applications of neural networks. Therefore, it is of prime importance to consider the effect of time delays on the dynamical behavior of neural networks. A large number of works on the stability of neural networks with time delays have been reported in the literature, see [6–21] and references therein. It is worthy pointing out that these delays mainly contain constant delays, time-varying delays, distributed delays and mixed time delays. However, it should be mentioned that very little attention was paid to neural networks with time delay in the leakage (or "forgetting") term, which is quite different from the above traditional time delays. It is known that this new class of delays is usually called leakage delays, which was originally introduced by Gopalsamy [22] in the study of neural networks. Generally speaking, the leakage delays often have a tendency to destabilize the neural networks, which was discussed in [22]. Moreover, it was pointed out in [23] that the leakage delays are difficult to handle. Thus, it is very interesting and challenging to investigate the stability of neural networks with time delays in the leakage term. More details on the leakage delays, we refer the reader to [24–29] and references therein.

On the other hand, Tsunoda and Nakamura in [30] showed that it is reasonable to investigate the mechanism of expectation on probabilistic stimuli through psychophysical experiments and analysis of the data given by a linear neuronal model. Based on the architecture of real systems, we should take expectations into coefficients of the model. It is inspiring that there have recently appeared a few works in the literature, which considered the expectation in coefficients. For instance, Hu and Huang in [31] studied a class of nonlocal stochastic differential equations, whose paths depend not only on their states but also on their

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expectations. By using the $M$-matrix technique, the authors in [32] discussed the existence and uniqueness as well as moment estimates of solutions for a class of nonlocal stochastic differential delay equations with time varying delay whose coefficients are dependent on expectations. Very recently, Hu investigated the mean square exponential stability for a class of stochastic Hopfield neural networks with expectations in coefficients. However, all of the above mentioned works with expectations in coefficients did not consider leakage delays. As mentioned before, the leakage delays have a great impact on the dynamics of neural networks. Therefore, it is an unsolved important problem to investigate the stability of stochastic neural networks with expectations in coefficients and time delays in the leakage term.

Motivated by the above discussion, in this paper we investigate the problem of mean square exponential stability for a class of stochastic neural networks with expectations in coefficients and time delays in the leakage term. We first establish a novel stability lemma of the suggested model. Then, by employing this new lemma, the Itô formula, the Lyapunov–Krasovskii functional, stochastic analysis theory and matrix inequalities technique, we derive some novel sufficient conditions to guarantee the mean square exponential stability of our considered model. Moreover, we have also reduced our model to the case of stochastic neural networks without leakage delays. Finally, an example is given to show the effectiveness of our results.

The rest of the paper is arranged as follows. In Section 2, we introduce the new model, some necessary assumptions and preliminary lemmas. In Section 3, we establish an important stability lemma and apply this new lemma to derive some novel sufficient conditions that guarantee the mean square exponential stability of our considered model. In Section 4, we provide an example to illustrate the effectiveness of the obtained results. Finally, we conclude the paper with some general remarks in Section 5.

## 2. Model formulation and preliminaries

In this section, we will introduce the model, some assumptions and preliminary lemmas.

We are interested in the following new class of stochastic neural networks with time delays in the leakage terms whose coefficients are dependent on expectations:

$$\begin{aligned} dx(t) = [&-Ax(t-\beta) - B\mathbb{E}x(t-\beta) + Cf(x(t), \mathbb{E}x(t)) \\ &+ Dg(x(t-\tau(t)), \mathbb{E}x(t-\tau(t)))]\, dt \\ &+ \sigma(t, x(t), x(t-\beta), x(t-\tau(t)), \mathbb{E}x(t), \mathbb{E}x(t-\beta), \mathbb{E}x(t-\tau(t)))\, d\omega(t), \end{aligned} \tag{2.1}$$

where $x = [x_1, x_2, \ldots, x_n]^T$ is the state vector associated with the $n$ neurons. $f = [f_1, f_2, \ldots, f_n]^T, g = [g_1, g_2, \ldots, g_n]^T$ are the neuron activation functions. $A$ and $B$ are positive diagonal matrices. $C = (c_{ij})_{n\times n}$ and $D = (d_{ij})_{n\times n}$ are the connection weight matrices. $\mathbb{E}[\cdot]$ stands for the correspondent expectation operator with respect to the given probability measure $P$. The diffusion coefficient $\sigma : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times m}$ is a Borel measurable function and satisfies the local Lipschitz condition, $\beta$ denotes the leakage delay and $\tau(t)$ is a time-varying delay. $\omega(t)$ is an $m$ dimensional Brownian motion defined on the complete probability spare $(\Omega, \mathcal{F}, P)$ with a natural filtration $\{\mathcal{F}_t\}_{t\ge 0}$, which satisfies the usual condition.

Denote that for any $t \ge 0$,

$$\Gamma(t) = t - \tau(t). \tag{2.2}$$

As usual, we assume that there exists a positive constant $\tau_1$ such that

$$0 \le \tau(t) \le \tau_1. \tag{2.3}$$

Moreover, we suppose that

$$\eta := \inf_{t\ge 0} \dot{\Gamma}(t) > 0, \tag{2.4}$$

which implies that there exists a positive constant $\tau_2$ such that

$$\dot{\tau}(t) \le \tau_2 < 1, \tag{2.5}$$

and

$$\eta \le \dot{\Gamma}(t), \tag{2.6}$$

for any $t \ge 0$. Obviously, $\Gamma(t)$ is strictly increasing and trends to infinite as $t$ trends to infinite.

Let $\tau = \max\{\beta, \tau_1, \tau_2\}$ and $C([-\tau, 0], \mathbb{R}^n)$ denote the family of continuous function $\xi$ from $[-\tau, 0]$ to $\mathbb{R}^n$ with the uniform norm: $\|\xi\| = \sup_{-\tau\le\theta\le 0}|\xi(\theta)|$, where $|\cdot|$ represents the Euclidean norm of vectors. Denote by $L^2_{\mathcal{F}_t}([-\tau, 0]; \mathbb{R}^n)$ the family of all $\mathcal{F}_t$ measurable, $C([-\tau, 0]; \mathbb{R}^n)$-valued stochastic variables $\xi = \{\xi(\theta) : -\tau \le \theta \le 0\}$ such that $\int_{-\tau}^{0} \mathbb{E}|\xi(\theta)|^2 d\theta < \infty$.

Throughout this paper, we suppose that the following conditions are satisfied.

**Assumption 2.1.** There exist non-negative constants $k_1, k_2, l_1, l_2$ satisfying

$$|f(x, y) - f(\overline{x}, \overline{y})| \le k_1|x - \overline{x}| + k_2|y - \overline{y}|,$$
$$|g(x, y) - g(\overline{x}, \overline{y})| \le l_1|x - \overline{x}| + l_2|y - \overline{y}|,$$

for all $x, \overline{x}, y, \overline{y} \in \mathbb{R}^n$.

**Assumption 2.2.** There exist positive definite matrices $T_1, T_2, T_3, T_4, T_5, T_6$ such that

$$\begin{aligned} &\mathrm{trace}[\sigma^T(x_1, x_2, x_3, x_4, x_5, x_6)\sigma(x_1, x_2, x_3, x_4, x_5, x_6)] \\ &\quad \le x_1^T T_1 x_1 + x_2^T T_2 x_2 + x_3^T T_3 x_3 + x_4^T T_4 x_4 + x_5^T T_5 x_5 + x_6^T T_6 x_6, \end{aligned}$$

for all $x_1, x_2, x_3, x_4, x_5, x_6 \in \mathbb{R}^n$.

**Assumption 2.3.** For any $t \ge 0$,

$$f(0, 0) = g(0, 0) \equiv 0, \quad \sigma(t, 0, 0, 0, 0, 0, 0) \equiv 0.$$

It is clear that under Assumptions 2.1–2.3, the local Lipschitz condition and linear growth condition hold. Therefore, it follows from [31] that there exists a unique global solution $x(t;\xi)$ to Eq. (2.1) for any initial data $\xi \in L^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$. In particular, Assumption 2.3 implies that Eq. (2.1) admits a trivial solution or zero solution $x(t;0) = 0$ when the initial condition $\xi \equiv 0$.

Next, let us state the definition of the exponential stability in the mean square to Eq. (2.1) as follows.

**Definition 2.4.** The trivial solution of Eq. (2.1) is said to be exponential stable in the mean square if for every $\xi \in L^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$, there exist scalars $\alpha > 0$ and $\gamma > 0$ such that the following inequality holds:

$$\mathbb{E}|x(t;\xi)|^2 \le \alpha e^{-\gamma t} \sup_{-\tau\le\theta\le 0} \mathbb{E}|\xi(\theta)|^2.$$

For the sake of simplicity, we let

$$\begin{aligned} F(t) = &-Ax(t-\beta) - B\mathbb{E}x(t-\beta) + Cf(x(t), \mathbb{E}x(t)) \\ &+ Dg(x(t-\tau(t)), \mathbb{E}x(t-\tau(t))), \end{aligned} \tag{2.7}$$

$$\Sigma(t) = \sigma(t, x(t), x(t-\beta), x(t-\tau(t)), \mathbb{E}x(t), \mathbb{E}x(t-\beta), \mathbb{E}x(t-\tau(t))), \tag{2.8}$$

for all $t \ge 0$.

Then, Eq. (2.1) is equivalent to

$$dx(t) = F(t)\, dt + \Sigma(t)\, d\omega(t).$$

Let $C^2(\mathbb{R}^n; \mathbb{R}_+)$ denote the family of all non-negative functions $V(t, x(t))$ which are continuously differentiable in $t$ and continuously