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Leaderless consensus of multi-agent systems with Lipschitz nonlinear dynamics and switching topologies

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ABSTRACT

This paper considers the leaderless consensus problem of multi-agent systems with Lipschitz nonlinearities. The communication topology is assumed to be directed and switching. Based on the property that the graph Laplacian matrix can be factored into the product of two specific matrices, the consensus problem with switching topologies is converted into a stabilization problem of a switched system with lower dimensions by performing a proper variable transformation. Then the consensus problems are solved with two different topology conditions. Firstly, with the assumption that each possible topology contains a directed spanning tree, the consensus problem is solved using the tools from stability analysis of slow switching systems. It is proved that the leaderless consensus can be achieved if the feedback gains matrix is properly designed and the average dwell time larger than a threshold. Secondly, by using common Lyapunov function based method, the consensus problem with arbitrary switching topologies is solved when each possible topology is assumed to be strongly connected and balanced. Finally, numerical simulations are provided to illustrate the effectiveness of the theoretical results.

1. Introduction

Recently, the consensus problem of multi-agent systems has drawn great attention for its broad potential applications in many areas such as cooperative control of vehicle, unmanned air vehicle formation and flocking control [1,2]. Consensus means that all agents will reach a common state in a cooperative fashion throughout distributed controllers. It is called leaderless consensus problem if there is no specified leader in the multi-agent systems, it is called leader-following consensus problem otherwise. Many results have been obtained [3–9]. Note that, all these results are obtained with fixed communication topologies. However, in many applications, the interaction topology among agents may change dynamically. This may happen when the communication links among agents may be unreliable due to disturbance or subject to communication range limitations [10].

Motivated by this, the consensus problems with switching communication topologies have been investigated in [11–17]. Under undirected jointly connected communication topologies, [11] solved the leaderless consensus problem of linear multi-agent systems using extended Barbalat's lemma. When each possible directed topology was balanced, leaderless consensus problem of linear multi-agent systems was solve with common Lyapunov function approach [12]. In [13], by using the multiple Lyapunov function approach, the leaderless consensus problem of linear multi-agent systems was solved with the assumption that each possible topology contained a directed spanning tree. In [14], H_∞ consensus problem of linear multi-agent systems with external disturbance was investigated with slow switching topologies. Based on averaging method, [15,16] solved the leader-following consensus problem of linear multi-agent systems with jointly connected topologies. Under the assumption that each possible topology had a directed spanning tree rooted at the leader, [17] solved the leader-following consensus problem of linear multi-agent systems with switching topologies and occasionally missing control inputs.

At the same time, the consensus problems of multi-agent systems with Lipschitz nonlinearities were investigated with switching communication topologies in [18–23]. In [18], the leader-following consensus problem of nonlinear multi-agent systems was considered with undirected and jointly connected topologies. With the assumption that each possible topology contains a directed spanning tree, the leader-following consensus problem was investigated in [19] with M-matrix theory and the tools from the stability analysis of switched system. In [20], the leader-following consensus problem was investigated with jointly connected topologies using distributed adaptive protocols. In [21], when there were randomly occurring nonlinearities and uncertainties and stochastic disturbances, the leader-following consensus problem was solved in the mean square sense. With the

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assumption that each possible topology contains a directed spanning tree, [22,23] solved the leaderless consensus of the first-order multi-agent systems with Lipschitz nonlinearities.

Note that, when the directed communication topologies are assumed to be time-varying and there are Lipschitz uncertainties in system dynamics, the results of the leaderless consensus problem are obtained with the system dynamics being restricted to be first-order [22,23]. As for high-order dynamics which can include the first-order dynamics as a special case, the existing conclusions are mainly focused on the leader-following case [19–21]. Considering the fact that, for the homogeneous agents, the leaderless consensus problem can include the leader-following consensus problem as special cases, the leaderless case is more challenging than the leader-following case. There are two main reasons. First, in the leader-following case, the consensus problem can be conveniently converted into a stabilization problem of a switched system by constructing the tracking error variables. Then the stability analysis method of switched system and the M-matrix theory can be adopted for analysis directly. As for the leaderless consensus problem, this is no specified leaders and M-matrix theory is not applicable to this case due to the singularity of the Laplacian matrix of the directed topology. Second, in the leader-following case, it is requited that each possible connected (or jointly connected) topology should have the same root. However, in the leaderless case, the roots of all possible topologies are not necessarily the same. This means that the requirement of the topology in the leaderless consensus problem is quite weaker than that in leader-following case. Actually, until now, when the communication topology is assumed to be directed and switching, the leaderless consensus problem of high-order multi-agent systems with Lipschitz nonlinearities has not been solved.

Motivated by above observation, this paper aims to solve the leaderless consensus problem of high-order multi-agent systems with Lipschitz nonlinearities and directed switching topologies. By performing a special kind of matrix decomposition, the graph Laplacian matrix is factored into the product of two specific matrices. Based on this property of the Laplacian matrix and a properly performed variable transformation, the consensus problem with switching topologies is converted into a stabilization problem of a switched system with lower dimensions. Then the leaderless consensus problem is solved with following two different topology conditions. Firstly, we assume that each possible topology contains a directed spanning tree. The consensus problem is solved with restricted switching topologies. The tools from stability analysis of slow switching systems are employed for analysis. It is proved that the leaderless consensus can be achieved if the feedback gains matrix is properly designed and the average dwell time is large than a threshold. Secondly, we assume that each possible topology is strongly connected and balanced. Then the consensus problem is solved with arbitrary switching topologies using the common Lyapunov function based approach.

In summary, the main contributions of the present work are two-fold. Firstly, the system dynamics of the agents is quite general, which can include the agents with first-order dynamics as special cases. Secondly, when the topologies are assumed to be directed and switching, the leaderless consensus problem is solved under two different topology conditions.

The remainder of this paper is organized as follows. In Section 2, some preliminaries and the problem formulation are provided. In Section 3, the leaderless consensus problem is solved with restricted switching topologies. In Section 4, the leaderless consensus problem is solved with arbitrary switching topologies. In Section 5, some simulation examples are presented. Section 6 is the conclusion.

2. Preliminaries and problem formulation

2.1. Preliminaries

In this paper, following notations will be used. $\mathbb{R}^{n\times n}$ and $\mathbb{C}^{n\times n}$ denote the set of $n\times n$ real and complex matrices, respectively. $\otimes$ denotes the Kronecker product. For $\mu\in\mathbb{C}$, the real part is $\mathrm{Re}(\mu)$. I_n is the $n\times n$ identity matrix. $\|\cdot\|$ stands for the induced matrix 2-norm. For a square matrix A, $\lambda(A)$ denotes the eigenvalues of matrix A; $rank(A)$ denotes its rank. The inertia of a symmetric matrix A is a triplet of nonnegative integers (m,z,p) where m, z and p are respectively the number of negative, zero and positive elements of $\lambda(A)$, $\max\{\lambda(A)\}$ $(\min\{\lambda(A)\})$ denotes the largest (smallest) eigenvalue of the matrix A. $A>B$ $(A\geq B)$ means that $A-B$ is positive definite (respectively, positive semidefinite). (A,B) is said to be stabilizable if there exists a real matrix K such that $A+BK$ is Hurwitz.

A directed graph $G=(V,\mathscr{E},A)$ contains the vertex set $V=\{1,2,\ldots,N\}$, the directed edges set $\mathscr{E}\subseteq\mathcal{V}\times\mathcal{V}$, the weighted adjacency matrix $\mathcal{A}=[a_{ij}]_{N\times N}$ with nonnegative elements a_{ij}. $a_{ij}=1$ if there is a directed edge between vertex i and j, $a_{ij}=0$ otherwise. The set of neighbors of i is defined as $N_i:=\{j\in V:\ a_{ij}=1\}$. A directed path is a sequence of ordered edges of the form $(i_1,i_2),(i_2,i_3),\ldots$, where $i_j\in\mathcal{V}$. The Laplacian matrix of the topology $\mathcal{G}$ is defined as $\mathscr{L}=[\mathscr{L}_{ij}]_{N\times N}$, where $\mathscr{L}_{ii}=\sum_{j\neq i}a_{ij}$ and $\mathscr{L}_{ij}=-a_{ij}$. Then 0 is an eigenvalue of $\mathscr{L}$ with 1_N as the eigenvector. A directed graph is called balanced if $\sum_{j=1}^{N}a_{ij}=\sum_{j=1}^{N}a_{ji}$. A directed graph is said to have a spanning tree if there is a vertex called the root such that there is a directed path from this vertex to every other vertex. A directed graph is said be strongly connected if there is a directed path between every pair of distinct vertices.

In this paper, the communication topology is molded by a directed graph and we assume that the communication topology is time-varying. Denote $\hat{\mathcal{G}}=\left\{\mathcal{G}^1,\mathcal{G}^2,\ldots,\mathcal{G}^p\right\},p\geq 1$ be the set of all possible directed topologies. We define the switching signal $\sigma(t)$, where $\sigma(t)$: $[0,+\infty)\rightarrow P=\{1,2,\ldots,p\}$. $0=t_0<t_1<t_2<\ldots$ denote the switching instants of $\sigma(t)$. Let $\mathcal{G}^{\sigma(t)}\in\hat{\mathcal{G}}$ be the communication topology at time t. Across each time interval $[t_j,t_{j+1})$, $j\in Z$, the graph $\mathcal{G}^{\sigma(t)}$ is fixed.

Lemma 1. *(Ren and Beard [10]). Zero is a simple eigenvalue of $\mathscr{L}$ and all the other nonzero eigenvalues have positive real parts if and only if the graph $\mathcal{G}$has a directed spanning tree, i.e., $0=\lambda_1<\mathrm{Re}(\lambda_2(\mathscr{L}))\leq\ldots\leq\mathrm{Re}(\lambda_N(\mathscr{L}))$.*

Lemma 2. *(Yu et al. [3]). Suppose that the graph $\mathcal{G}$ is strongly connected and balanced. Then, $\mathscr{L}+\mathscr{L}^T$ is positive semi-definite with zero being its simple eigenvalue.*

2.2. Problem formulation

Consider a multi-agent system composed of N agents with the following identical dynamics

$$\dot{x}_i(t)=Ax_i(t)+Bu_i(t)+Df(x_i(t),t)\quad i=1,2,\ldots,N, \tag{1}$$

where $x_i(t)\in\mathbb{R}^n$, $u_i(t)\in\mathbb{R}^p$are the state and the control input of the $i-$th agent , respectively. A, B and D are constant system matrices with compatible dimensions. $f:\mathbb{R}^n\times[0,\infty)\rightarrow\mathbb{R}^m$ is a continuously differentiable vector-valued function representing the nonlinearities which satisfies Lipschitz condition, i.e.,

$$\|f(x(t),t)-f(y(t),t)\|\leq\rho\|x(t)-y(t)\|,\quad\forall x,y\in\mathbb{R}^n,t\geq 0,$$

where $\rho>0$ is a constant scalar.

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