Omega 41 (2013) 406–420

Contents lists available at SciVerse ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

Portfolio rebalancing with an investment horizon and transaction costs

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ARTICLE INFO

Article history:
Received 1 March 2011
Accepted 9 March 2012
Available online 10 May 2012

Keywords:
Portfolio optimisation
Transaction cost
Efficient frontier
Portfolio rebalancing
Multiperiod portfolio optimisation

ABSTRACT

In this paper we consider the problem of rebalancing an existing financial portfolio, where transaction costs have to be paid if we change the amount held of any asset. These transaction costs can be fixed (so paid irrespective of the amount traded provided a trade occurs) and/or variable (related to the amount traded). We indicate the importance of the investment horizon when rebalancing such a portfolio and illustrate the nature of the efficient frontier that results when we have transaction costs. We model the problem as a mixed-integer quadratic programme with an explicit constraint on the amount that can be paid in transaction cost. Our model incorporates the interplay between optimal portfolio allocation, transaction costs and investment horizon. We indicate how to extend our model to include cardinality constraints and present a number of enhancements to the model to improve computational performance. Results are presented for the solution of publicly available test problems involving up to 1317 assets.

1. Introduction

In forming a portfolio of financial assets (such as stocks, equities) the basic approach adopted derives from the well-known work of Markowitz [41] who considered the problem as one of trading off reward (as measured by mean portfolio return) against the risk involved (as measured by variance in portfolio return). The approach he proposed is now well-known and often seen in graphical form (an efficient frontier, with return being plotted against risk). In this paper we will assume that the reader is familiar with the basic approach adopted in Markowitz mean–variance portfolio optimisation.

In this paper we consider the problem of rebalancing an existing financial portfolio, where transaction costs have to be paid if we change the amount held of any asset. These transaction costs can be fixed (so paid irrespective of the amount traded provided a trade occurs) and/or variable (related to the amount traded). Introducing transaction costs into a Markowitz framework, effectively incurring a financial penalty for trading, means that the investment horizon over which we will hold the rebalanced portfolio unchanged is important. This contrasts with the basic Markowitz model where the investment horizon is effectively irrelevant, since in that model there are no transaction costs and so no penalty associated with trading.

In this paper we introduce fixed and variable transaction costs into a multiperiod Markowitz framework. We illustrate the nature of the efficient frontier that results when we have transaction costs. Our model of the problem is a mixed-integer quadratic programme with an explicit constraint on the amount that can be paid in transaction cost. The model can be extended to include cardinality constraints, plus a number of other restrictions that are of relevance in practical portfolio optimisation. Computational results are presented in this paper for the solution of problems involving up to 1317 assets.

In general terms this paper addresses a financial problem via mathematical modelling and optimisation. This is a common theme in the literature (e.g. see [4,17,43,44] for work as to this). We make use of the Markowitz framework which, whilst well-known for financial problems, also has applications in other areas (e.g. see [2,39]).

This paper is structured as follows. In Section 2 we motivate our approach and briefly review the literature on portfolio optimisation where transaction costs are involved. In Section 3 we present our formulation of the problem indicating how we compute portfolio return and risk in a multiperiod (investment horizon) setting. We extend our formulation of the problem to include cardinality constraints and present a number of computational enhancements. In Section 4 we present computational results for publicly available test problems illustrating the effectiveness of our approach and the efficient frontiers that typically result. In Section 5 we present our conclusions.

2. Motivation, contribution and literature review

In this section we motivate our approach by means of a small example and state what we believe to be the contribution of this

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0305-0483/$ - see front matter © 2012 Published by Elsevier Ltd.
http://dx.doi.org/10.1016/j.omega.2012.03.003

paper. We then go on to (briefly) review the literature relating to portfolio optimisation where transaction costs are involved.

2.1. Example

In standard Markowitz analysis we take single period returns and look at their means, variances and covariances. But when transaction costs are present we also have to consider the investment horizon. To illustrate this suppose we have \$100 in cash and (for simplicity) suppose we have just two possible portfolios in which we can invest:

- a portfolio of value \$75 which gives a mean return of 3% per period (\$25 having being consumed in transaction cost in purchasing the portfolio),
- a portfolio of value \$50 which gives a mean return of 5% per period (\$50 having being consumed in transaction cost in purchasing the portfolio).

Clearly the second portfolio has the higher return, but our initial investment in that portfolio is less after accounting for the transaction cost paid on our initial investment of \$100. Utilising compound interest over time the two portfolios will have an equal (expected) value after an investment horizon H which satisfies $75(1.03)^H = 50(1.05)^H$. This is a simple equation to solve which yields $H = 21.08$. Hence, unless we are investing for (approximately) 21 periods or more, the portfolio with a higher return will have a lower terminal value at the end of the investment horizon. Note here that this argument applies irrespective as to how the transaction cost incurred is calculated (purely fixed; purely variable; mix of fixed and variable).

We can regard our initial cash of \$100 as the portfolio that we are currently holding. Should we therefore change (rebalance) from this current portfolio to either of the two portfolios considered above? The length of the investment horizon is important not just in deciding between two portfolios (as above). It also inputs into the decision as to whether to invest (rebalance) or not. For simplicity suppose our \$100 in cash earns no interest. Then it is not worthwhile investing in the first portfolio if $75(1.03)^H < 100$, i.e. if $H < 9.73$; it is not worthwhile investing in the second portfolio if $50(1.05)^H < 100$ i.e. if $H < 14.21$.

Taken together we have that if $H < 9.73$ we retain our initial cash portfolio (so do not rebalance); if H lies between 9.73 and 21.08 we rebalance to the portfolio with 3% return; if $H > 21.08$ we rebalance to the portfolio with 5% return.

There is a further issue that needs mention here. When a decision-maker decides to invest in (or rebalance to) a portfolio they may have in mind what they view as an minimum acceptable return (e.g. perhaps derived from the risk-free rate or the return available from other opportunities). Taking our example above if this minimum acceptable return is 4% (say) then the portfolio offering a 3% return will never be acceptable (irrespective of the value of transaction cost and irrespective of the length of the investment horizon). Only when the return from the portfolio exceeds the minimum acceptable return does the investment horizon become relevant. In such cases we need time for the portfolio to recover from the loss in value due to transaction cost and reach the minimum acceptable return level. For example here the time need for our initial cash of \$100 to reach a 4% return if we spend \$50 in transaction cost and invest \$50 in the portfolio offering 5% return is given by the solution to $100(1.04)^T = 50(1.05)^T$, i.e. $T = 72.43$. Hence we would need an investment horizon $H \geq 72.43$ to consider it being worthwhile investing.

In summary here the length of the investment horizon plays a key role, in deciding if (and how) to rebalance a portfolio. Note here that any decision-maker may well not have a pre-determined investment horizon. Rather they may (for example) be prepared to adopt a longer investment horizon than they were initially considering if the return they receive (for the risk they take) is better than they anticipated. The model we present below enables us to see, in a graphical fashion, the tradeoff that occurs between return, risk and investment horizon.

The contribution of this paper is that the model we present incorporates the interplay between optimal portfolio allocation, transaction costs and investment horizon. It is, to the best of our knowledge, the first paper in the literature to combine these factors together in a single mean–variance model without any underlying assumptions as to asset price/return dynamics. Although other authors in the literature (as considered below) have included transaction costs they typically deal with just a single period horizon. As far as we are aware there are no models in the literature that are similar to our multiperiod horizon model.

2.2. Literature review

In this section we review the literature relating to previous work for portfolio optimisation where transaction costs are involved (either in creating an initial portfolio from cash or in rebalancing an existing portfolio). Because there is a large volume of such work (and because this paper is primarily directed towards presenting a new model, not discussing previous work) we only consider in detail selected papers from recent years (2006 onwards). Earlier papers are referenced, but not discussed. Additional papers of which we are aware include [1,20,21,23,26,27,45–48,55–57]. We have excluded from our review work based on assuming that the stochastic process underlying asset price/return dynamics is known.

In this paper we define a fixed cost as being associated with making a trade (i.e. a transaction) in an asset. This contrasts with some papers in the literature (e.g. [21,27,45]) where fixed cost is defined slightly differently, for example as related to the presence or not, of an asset in the portfolio. These two definitions coincide if we are creating a portfolio from cash (a portfolio ***creation*** problem). However they are different if, as in the problem considered in this paper, we are ***rebalancing*** an existing portfolio, where we can hold an asset in both the current and rebalanced portfolio without doing any trading in the asset.

Common assumptions in the literature as to variable transaction cost are: a V-shaped function, where transaction cost is directly related to the absolute difference between the proportion (or number of units) associated with an asset in the new and current portfolio; or a concave function, where transaction cost is a concave function of the amount traded.

2.2.1. No fixed transaction cost: V-shaped variable transaction cost function

Bertsimas and Pachamanova [10] presented an approach based upon robust optimisation [8] where a cash account is used to account for asset trades (so for example selling some of the current holding of an asset means that the net amount, after transaction cost, is added to the cash account). Computational results were presented for problems involving three and 25 assets. Best and Hlouskova [15] modified an active set quadratic programming algorithm. Computational results were given for randomly generated problems involving up to 500 assets. Yu and Lee [54] presented a number of multi-criteria models, some incorporating skewness and kurtosis, that were solved using fuzzy multi-objective programming. Computational results were given for problems involving 45 assets. Relevant earlier work on this theme of a V-shaped cost function can be found in [11,12,37,38,51,53].