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Discriminating thresholds as a tool to cope with imperfect knowledge in multiple criteria decision aiding: Theoretical results and practical issues

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ABSTRACT

This paper deals with preference modeling. It concerns the concepts of discriminating thresholds as a tool to cope with the imperfect nature of knowledge in decision aiding. Such imperfect knowledge is related with the definition of each criterion as well as with the data we have to take into account. On the one hand, we shall present a useful theoretical synthesis for the analyst in his/her decision aiding activity, and, on the other hand, we shall provide some practical instructions concerning the approach to follow for assigning the values to these discriminating thresholds.

1. Introduction

Research on ordered structures requiring the definition of one or several thresholds gave birth to a wide range of theoretical works, as for instance, Krantz [14], Luce [16], Cozzens and Roberts [7], Suppes et al. [31], Vincke [34], Abbas and Vincke [1], Pirlot and Vincke [19], Tsoukiàs, and Vincke [32], Ngo The and Tsoukiàs [18].

The ordered structures with one or two thresholds are of a particular interest in decision aiding for modeling the imperfect knowledge [4,8,12,23,28,29,30].

Preference modeling in decision aiding needs to take adequately into account the imperfect knowledge, especially in the case of multiple criteria methods (see, for instance, and concerning only Omega Journal [2,3,15,37]). Indeed, the definition of each criterion frequently comprises some part of arbitrariness, and the data used to built criteria are also very often imprecise, ill-determined, and uncertain. This is why, for instance:

(i) In the definition of a net present value, the elements to be taken into account (the amortization period and the discount rate) lead to make some choices, which comprise a part of *arbitrariness*.
(ii) A criterion may be built from data obtained after a survey (through the application of questionnaires), which comprises inevitably an *imprecision* margin.
(iii) As soon as certain data (parameters), to take into account in a given criterion, are represented by the values these parameters will possess in a more or less distant future, we are in the presence of an *uncertainty*, which may be important.
(iv) Certain types of consequences or outcomes that must be taken into account by a given criterion are difficult to define. They are *ill-determined*. This is the particular case of the market share conquered by a company, the quality of a product, the degree of inconvenience of a population due to a noise nuisance. Provide precise definitions for these concepts are a very hard and frequently impossible task.

There are several decision aiding models and method that make use of the concept of thresholds for modeling this imperfect knowledge; they may use one or two thresholds, called *discriminating thresholds* [5,17,20,22,24,35].

After bringing to light, in Section 2, the interest and the role of the concept of discriminating thresholds in decision aiding, we shall define formally, in Section 3, the concept of pseudo-criterion by pointing out the existence of a double definition of the thresholds (direct and inverse) and by giving the relation between both (see Theorem 1). Then, we shall present, in Section 4, a synthesis of the main theoretical results in decision aiding. We shall devote an extended section, Section 5, to the way the analyst should proceed in practice to assign adequate values to these thresholds and this for a variety of possible contexts.

Our main concern in this paper is to call the attention of the reader to the pitfalls that can come from the difference between direct and inverse thresholds with respect to a criterion to be

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maximized or a criterion to be minimized, or from the discrete or continuous nature of the scale, especially within the framework of the Electre methods [9,10,36].

2. Discriminating thresholds in decision aiding: For what purpose?

In this section we present some preliminary concepts and illustrate the purpose of making use of discriminating thresholds in decision aiding through four pedagogical examples.

2.1. Preliminary concepts

In what follows A denotes a set of *potential actions*. Each action, $a \in A$, can be defined by a brief descriptive phrase or term, say a label, corresponding to an extensive description. In such a case, A can be defined as follows, $A=\{a_1, a_2, \ldots, a_i, \ldots\}$. This set can be completely known *a priori* or it may also appear progressively during the decision aiding process. The actions a can also be elements of $\mathbb{R}^m$; they may represent solutions of a feasible set defined through mathematical constraints. In such a case, A is a set containing elements a of $\mathbb{R}^m$. Let g denote a given *criterion*, built for characterizing and comparing potential actions according to a considered point of view. This characterization of an action $a \in A$, denoted by $g(a)$, usually represents the *performance* of action a according to the point of view considered.

Let E_g denote the set of all possible performances, which can be assigned to actions $a \in A$ according to criterion g. Each element of E_g can be characterized by a pictorial object, a verbal statement, or more generally by a number. As for defining a preference model, E_g must be a completely ordered set: $>_g$ will be used to denote this order. When $>_g$ corresponds to the direction in which preferences increase, we say that g is a criterion to be *maximized*; in the opposite case, g is to be *minimized*. The completely ordered set E_g is called the *scale* associated with criterion g. The elements of the scale E_g are called *scale levels* or simply *levels*. The scale can be defined either by a sequence of *ordered levels* (discrete scales, see Examples 1 and 2, below) or by an interval of real numbers $[e_*, e^*]$ (continuous scales, see Examples 3 and 4, below). In practice, the scale is never really continuous since only certain rational numbers of the above interval are used to define a performance. The levels of a continuous scale are necessarily characterized by numerical values, while the different levels of a discrete scale can also be characterized by verbal statements. In such a case and since E_g is a completely ordered set, each level can again be characterized by a numerical value: its position or rank in the scale. In such conditions, $e_*=1$ is the lowest level, while $e^*=|E_g|=n$ represents the highest level on the scale E_g.

Defining a criterion g is to build and to choose an *operational instruction* able to associate with any action $a \in A$ a performance $g(a)=e \in E_g$ judged appropriate to compare any ordered pair of actions from the point of view of the considered criterion. This operational instruction can be, depending on the circumstances or cases, based on expert judgements, questionnaires, forecasting techniques, several measurement tools, mathematical expressions, or even more complex algorithms using multiple data. If this operational instruction is, in its very nature, enough devoid of ambiguity, subjectivity, and arbitrariness and if the data that it makes use of are enough reliable, then the criterion g, thus built, is a preference model, which can be considered legitimate to lead to the following conclusions:

(i) the *indifference* between two actions a and a' ($aI_g a'$) is established if and only if $g(a)=g(a')$;
(ii) the *preference* in favor of a over a' ($aP_g a'$) is established without ambiguity if and only if $g(a)>g(a')$ when the criterion is to be maximized and $g(a)<g(a')$ when the criterion is to be minimized (this is valid even for a very small performance difference separating $g(a)$ from $g(a')$).

The above preference model, defined by (i) and (ii), is called the *true-criterion* model. Very often, this model is not realistic. This missing of realism may come, as it will be explained through the four examples in next subsection, from different reasons: the operational instructions can incorporate some part of ambiguity, subjectivity, and arbitrariness. It can be supported by poor or fragile working hypotheses due to an imperfect knowledge of what we want to evaluate. These operational instructions can also make use of data obtained from imprecise measures or based on less rigorous definitions, or even data obtained from the application of less reliable procedures.

2.2. Some examples

In this subsection, four examples are presented aiming to illustrate the different concepts needed in the rest of the paper.

Example 1. Implementation time in number of months

$E_g=\{6, 7, \ldots, 35, 36\}$ (g is a criterion to be minimized)

Here, an action a is an investment project: the time we are interested in is the one that was estimated for being necessary to implement a project (viewed as a set of tasks). This estimation can neither be made with a precision of one month nor it can even probably be made with a precision of two months. This leads to suppose that:

(i) if two actions a and a' are such that $|g(a)-g(a')|=1$, then this performance difference is not significant;
(ii) to be able to conclude that the implementation time of a is significantly shorter than the implementation time of a', it is necessary to consider $g(a)<g(a')+2$.

In such conditions criterion g is a preference model that seems legitimate to support the following conclusions:

(i) the indifference $aI_g a'$ is established if and only if $|g(a)-g(a')| \leqslant 1$;
(ii) the preference $aP_g a'$ is established without ambiguity if and only if $g(a)<g(a')$ and $|g(a)-g(a')|>2$.

These conclusions are different from those provided by a true-criterion model. Moreover, they should be completed: what should we conclude in the case where $g(a)=g(a')-2$? This performance difference is clearly incompatible with $a'P_g a$. Nevertheless, this difference is considered very weak to lead us to unquestionably suppose that the implementation time of a is significantly lower than the implementation time of a'. In other words, we are in the presence of an ambiguity situation corresponding to a hesitation between the two conclusions, $aI_j a'$ and $aP_g a'$. If there is a preference it should be in favor of a over a', but such a preference is very weakly established to exclude by itself the possibility of an indifference between the two actions. This situation corresponds to what is called in decision aiding *weak preference* (*i.e.*, a weakly established preference) and denoted by $aQ_g a'$.

Example 2. Fitness with respect to an objective

$E_g=\{$opposing, neutral, possibly favorable but questionable, unquestionable but weak, significant but partial, complete$\}$
(g is a criterion to be maximized)

Criterion g should take into account the way different projects $a \in A$ make their contribution to an objective we assume well defined.