Robotics and Autonomous Systems 62 (2014) 889–909

Contents lists available at ScienceDirect

Robotics and Autonomous Systems

journal homepage: www.elsevier.com/locate/robot

Bifurcations and chaos in passive dynamic walking: A review

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HIGHLIGHTS

- The chaos research in Passive Dynamic Walking (PDW) is covered in the reviewed literature.
- An account of chaos control techniques in PDW bipeds is presented. This area certainly necessitates further investigation.
- The need of new mathematical methods is emphasized so that PDW bipeds can be studied analytically.
- Potential research directions have been identified.

ARTICLE INFO

Article history:
Received 25 June 2013
Received in revised form
31 December 2013
Accepted 13 January 2014
Available online 2 February 2014

Keywords:
Bifurcation
Bifurcation diagram
Chaos
Dynamics
Dynamical systems
Limit cycle
Passive dynamic walking

ABSTRACT

Irrespective of achieving certain success in comprehending Passive Dynamic Walking (PDW) phenomena from a viewpoint of the chaotic dynamics and bifurcation scenarios, a lot of questions still need to be answered. This paper provides an overview of the previous literature on the chaotic behavior of passive dynamic biped robots. A review of a broad spectrum of chaotic phenomena found in PDW in the past is presented for better understanding of the chaos detection and controlling methods. This paper also indicates that the bulk of literature on PDW robots is focused on locomotion on slope, but there is a thriving trend towards bipedal walking in more challenging environments.

1. Introduction

Passive Dynamic Walking (PDW) originated from the ability of the biped to act as an inverted-jointed pendulum during the single support phase. A passive biped robot walks by the dynamics only. Human-like walking and high efficiency are its two vital benefits. McGeer coined the term PDW and its principles in the early 1990s. He showed that an uncontrolled and unpowered biped robot could walk stably on shallow slopes if suitable initial conditions are applied [1–3].

Fig. 1 shows McGeer's passive bipedal machine—Dynamite. Its gait is generated automatically by gravity and inertia, without planning trajectory beforehand. This periodic gait is called the "limit cycle" in terms of dynamical system theory. The organic evolution of limit cycle is a key characteristic of passive dynamic walking that imitates human walking—which is controlled by the neuro-muscular system, i.e., it is powered by muscles and controlled by the nervous system. Without nerves and muscles, this complex and controlled process can be modeled as an unpowered (passive) mechanical process. The better insight of human locomotion, development of cutting-edge prosthetic limbs and superior design of humanoid robots is the impetus for the investigation in PDW robots.

Passive bipeds are dynamical systems which are sensitive to initial conditions due to which they exhibit chaotic behavior. Several chaotic dynamics, e.g., bifurcation, intermittency and crises, etc., occur in consequence of variation in different parameters in the dynamical equations of many simple passive biped models. Many researchers from diverse fields like engineering, mathematics, biomechanics and computer science have been fascinated by these erratic dynamics. The chaos research in PDW bipeds has emerged as an interdisciplinary research area and it could be useful in diagnosing gait pathologies [4].

The chaos research in PDW has resulted in a tectonic shift in the way PDW phenomena is viewed now. This article describes the

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http://dx.doi.org/10.1016/j.robot.2014.01.006
0921-8890/

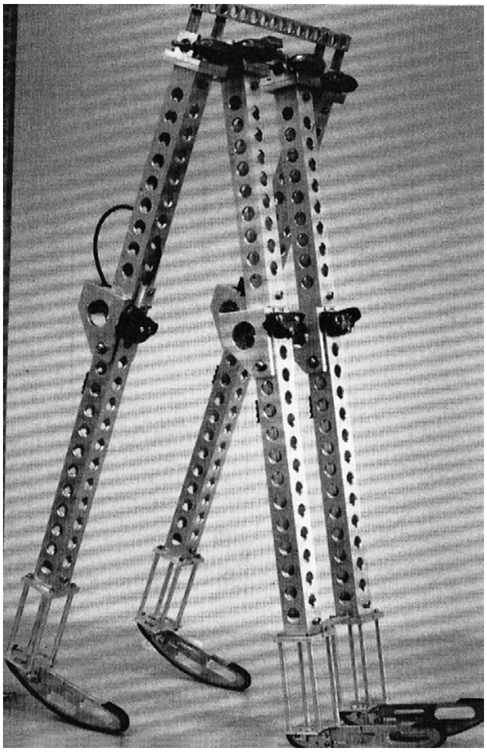

Fig. 1. McGeer's experimental passive biped [2].

current status and future of nonlinear dynamics study in passive walking. This paper is organized as follows: Section 2 introduces major terms of the chaos theory. In chronological order, the chaos exploration in passive bipeds is covered in Section 3. Similarly, Section 4 presents chaos control research in PDW sequentially and Section 5 discusses potential research directions in this research area. Section 6 gives concluding comments.

2. Chaotic dynamics: important terms

"Chaos" or "Deterministic Chaos" is aperiodic long-term behavior in a deterministic dynamical system that exhibits sensitive dependence on initial conditions. This sensitive dependence on initial conditions – that ensures uncertainty and unpredictability in the system behavior – is often called the "Butterfly Effect". A tiny perturbation to the initial conditions significantly alters the long term system dynamics. The Butterfly effect is the trademark of chaos theory. The chaos theory is an interdisciplinary area of study including mathematics, physics and engineering. This field investigates the dynamical behavior of systems which are extremely susceptible to initial conditions. It is applied in various scientific fields e.g., biology, economics, finance, geology, meteorology, population dynamics, psychology, and robotics. Since the advent of digital computers, chaos theory has become more comprehensible and is growing quickly. A glossary of important terms of this theory is included for new researchers [5–7]:

Bifurcation or branching is a sudden qualitative change in the dynamics as a system parameter varies. The parameter value at which branching occurs is called "Bifurcation Point," at which the system is unstable structurally. Technically, bifurcations are very vital as they provide paradigms of instabilities and transitions as some control parameter is modified. In both continuous and discrete systems bifurcations occur.

Bifurcation diagram is a chart which illustrates the steady-state behavior of a system over a range of parameter values. That is, it depicts the periodic orbits or fixed/equilibrium/critical points of a

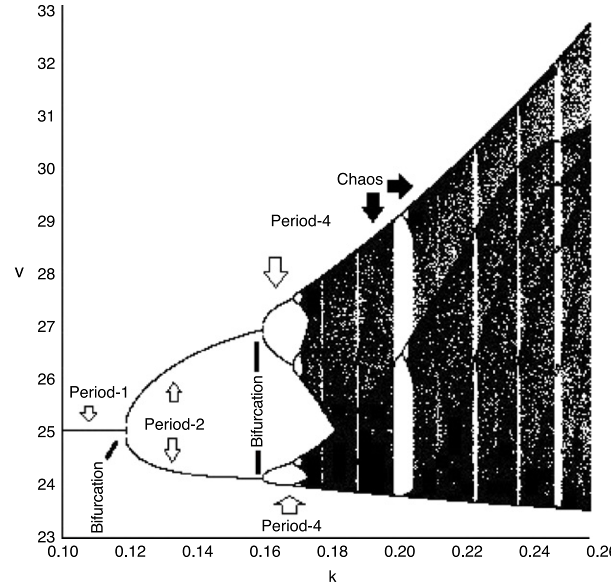

Fig. 2. Bifurcation diagram of a DC–DC buck converter [8].

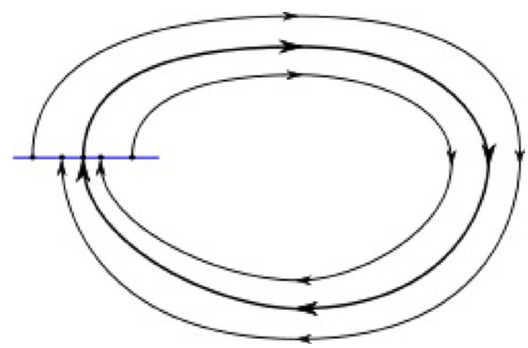

Fig. 3. Limit cycle.

system versus the bifurcation parameter as illustrated in Fig. 2. This graph summarizes the enormous information concisely so that system behavior as a function of a control (bifurcation) parameter can be observed. Fig. 2 shows the bifurcation diagram for a DC–DC buck converter [8].

Attractor is a set of points to which all adjacent trajectories converge as the number of iterations approaches to infinity i.e., points that get close enough to the attractor stay close even if slightly perturbed. It is a region in n-dimensional space. Since a trajectory can be periodic or chaotic, its instances are stable limit cycle, stable fixed point, and even an intricate strange attractor.

Limit cycle is an isolated periodic solution or orbit in the autonomous system as demonstrated in Fig. 3. Limit cycles are approached by nearby trajectories. They are categorized as "attractive or stable limit cycles" and "un-attractive or unstable limit cycles".

Strange/chaotic attractor is an attractor of a dissipative system that has noninteger (fractal) dimension. It is also called "fractal attractor". This shows sensitive dependence on initial conditions. Fig. 4 shows a strange attractor.

Basin of attraction also called "attracting basin", is the set of all initial conditions that leads trajectories to a given attractor.

Lyapunov exponent quantifies the rate of divergence of infinitesimally nearby trajectories starting from close initial conditions. The largest one is called "Maximal Lyapunov Exponent (MLE)" and a positive MLE usually indicates a chaotic attractor.