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Generalized symmetric weight assignment technique: Incorporating managerial preferences in data envelopment analysis using a penalty function

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ABSTRACT

In this paper, we generalize the Symmetric Weight Assignment Technique to incorporate all managerial preferences in Data Envelopment Analysis (DEA). This is a method that promotes managerial preferences, while not changing the feasibility region of the nominal DEA model, unlike standard techniques such as cone ratio and assurance region that potentially yield infeasible problems. We discuss how this generalization is motivated by a real word problem where the decision maker's preference is not precisely known. We also discuss how we are using our generalization in solving a workforce allocation problem for the United States Navy.

1. Introduction

Data Envelopment Analysis (DEA) is used in evaluating entities, commonly referred to as decision making units (DMUs), based on how well each entity transforms inputs to outputs. If there is a single input and a single output, then the evaluation is simply the ranking, largest to smallest, of the output to input ratio. Such a myopic approach to determining DMU efficiency does not work in the case of multiple inputs and multiple outputs. Instead, a linear programming approach is used to determine the efficiency of each DMU. This approach allows each DMU to evaluate its efficiency by considering a subset of inputs and a subset of outputs relative to all other DMUs (these subsets could be any non-empty subset of the inputs and outputs respectively).

One could imagine the case of a Factory X making widgets A and B that combine to make widget C. It is possible for Factory X to be the most efficient factory making widget A, but the most inefficient at making widget B. According to nominal DEA models, Factory X could be as efficient as a factory that is efficient at producing both widget A and B and a factory that is efficient at producing only widget B. From a managerial perspective, there is a clear need to differentiate factories that are seemingly efficient according to nominal DEA models, and factories that are efficient at producing an entire product. To this end, there are multiple proposed methods for promoting a managerial preference structure using DEA. The one approach we will extend in this paper is the Symmetric Weight Assignment Technique (*SWAT*) proposed by Dimitrov and Sutton [1]. *SWAT* evaluates DMUs by not only considering their efficiency, how they transform inputs to outputs, but also their ability to adhere to a symmetric managerial preference structure. As *SWAT* takes into account two components in evaluating a DMU, it does not generate a pure efficiency score as nominal DEA models (one such nominal model is shown in (1)). The resulting metric of a *SWAT* model is referred to as a *SWAT score*, which is a combination of efficiency and adherence to a symmetric managerial preference.

The purpose of this paper it to generalize *SWAT*, to the generalized Symmetric Weight Assignment Technique (*g-SWAT*) used to incorporate all managerial preferences, not just symmetric weight preferences. Similar to *SWAT*, the resulting metric of a *g-SWAT* model is referred to as a *g-SWAT score*, which is a combination of efficiency and adherence to a general managerial preference.

In this paper, we show how to generate *g-SWAT* from *SWAT*. We discuss how this generalization is non-trivial and leads to new behaviors not explored when *SWAT* was originally proposed. We then discuss how we are applying *g-SWAT* to an assignment problem for the United States Navy. Using *g-SWAT*, we assign sailors to jobs and measure how the resulting assignments perform on a set of metrics defined by the Navy. The Navy does not explicitly know how much one metric should be valued relative to other metrics, but would like to understand how assignments change as preferences vary. The *g-SWAT* method is developed for such a setting.

2. Background research

In this section we review some of the related work in promoting managerial preferences in DEA.

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Charnes et al. [2] originally developed DEA as a method to measure the relative efficiency of a group of similar decision making units. The original DEA formulation presented by Charnes et al. is given in (1). The linear program in (1) is solved once for every decision making unit. For a DMU, the optimal objective function value of (1) is its efficiency score. *Efficient DMUs* have a score of 1, and *inefficient DMUs* have a score greater than 1. For each inefficient DMU, a set of efficient DMUs are given as a *comparative set* for benchmarking that can be used as a target for improvement for the inefficient DMU:

$$\begin{aligned} \min_{\boldsymbol{p},\boldsymbol{q}} \quad & \boldsymbol{p}\boldsymbol{x_0} \\ \text{s.t.} \quad & \boldsymbol{q}\boldsymbol{y_0} = 1 \\ & -\boldsymbol{p}X + \boldsymbol{q}Y \le 0 \\ & \boldsymbol{p} \ge 0,\ \boldsymbol{q} \ge 0 \end{aligned} \tag{1}$$

In (1) and all subsequent DEA models, X is a matrix of the inputs with each column representing a DMU and each row representing an input dimension. Y is a matrix of the outputs, and just as X, each column represents a DMU and each row represents an output dimension. $\boldsymbol{x_0}$ and $\boldsymbol{y_0}$ are columns of X and Y respectively, and are the inputs and outputs of the DMU that is currently being evaluated by (1). $\boldsymbol{p}$ and $\boldsymbol{q}$ are vectors and the decision variables in the model, and they are the weights put on the input and output dimensions of the DMUs respectively.

Notice that the weights selected by a DMU must be feasible for all other DMUs and be greater than zero. This allows for what is called *free selection of weights*, which is not always practical for various applications. Often the decision maker has expert knowledge on the range of values the weights should take, or at least the relative magnitudes of the weights. Thus, DEA models have been constructed to account for the inclusion of this expert knowledge. This often means restricting the values of the weights. Models restricting weight values have been applied in various settings from evaluating nursing homes to city quality of life [3–5]. DEA weight restriction methods include cone ratio, assurance regions, and direct weight restrictions. However, all of these methods may lead to an infeasible problem depending on the bounds determined by an expert as discussed by Sarrico and Dyson [6] and Estellita Lins et al. [7].[1] Each of these methods are detailed below.

Other methods to include user preferences involve changing the comparative set of DMUs for inefficient DMUs. These methods are not covered in this paper because they are sufficiently different than the methods presented here. However, additional information on these methods is found in Olesen and Petersen [8] and Bessent et al. [9].

2.1. Assurance region/cone ratio

There are two types of constraints that are associated with assurance region or cone ratio models. The first set of constraints is meant to restrict the inputs or outputs relative to themselves. This is expressed in two forms as seen in (2). The first form, (2a), constrains the ratio of inputs or outputs weights by an upper and a lower bound, presented as λ_i, ϕ_i, τ_j, and ω_j. The second form of the constraint, (2b), restricts the weight on a particular input or output weight to be greater than the conical combination of the weight on two other inputs or outputs respectively. These types of constraints are very useful when the units of the outputs or inputs are the same. However, when the units are different the intuitive meaning of the constraints can be difficult to interpret:

$$\lambda_i \le q_i/q_{i+1} \le \phi_i, \quad \tau_j \le p_j/p_{j+1} \le \omega_j \tag{2a}$$

$$\kappa_i q_i + \kappa_{i+1} q_{i+1} \le q_{i+2}, \quad \psi_j p_j + \psi_{j+1} p_{j+1} \le p_{j+2} \tag{2b}$$

A second set of assurance region constraints involve a comparison of input weights to output weights as seen in (3). Though initially it may not be clear why the input and output weights should be tied together, Thompson et al. [10–12] show how these types of constraints can be used to evaluate relative profit and absolute profit, where p_j is the cost weight of input j and q_i is the profit weight of output i. This has led to a profit ratio form of DEA that is used in several applications such as coal mining, banking, parental care, *etc.* [11,13–15].

$$\gamma_i q_i \ge p_j \tag{3}$$

2.2. Direct weight restriction

One of the most popular ways to include preferences into DEA is through the use of weight restrictions. This allows for a direct restriction on the allowable values that weights can assume, and potentially pruning the feasibility region. The simplest of these methods is direct weight restriction which places a upper and lower bound on the weight. These inequalities take the form shown in the following equation:

$$\begin{aligned} &\eta_i \le q_i \le \delta_i \\ &\nu_j \le p_j \le \mu_j \end{aligned} \tag{4}$$

One of the shortcomings of direct weight restriction is the implicit meaning of the bounds η_i,δ_i,ν_j, and μ_j. Since these bounds are dependent on the units of the outputs, they have little meaning besides their relative magnitude. In Podinovski [16], the author makes the argument that direct weight restriction can also change the basic concept of maximum relative efficiency. This could produce serious concerns about the validity of the weights selected and reduce the usefulness of direct weight restriction.

2.3. Relationship between fuzzy DEA and g-SWAT

It is important to highlight the difference between *g-SWAT* and existing weight restriction methodologies. In the above methods, a decision maker precisely knows her preference structure, *i.e.*, absolute or relative weight range that the DEA decision variables must take. On the other hand, *g-SWAT* is primarily used in situations where preferences are not precisely known. This can be likened to instances where data elements, the X and Y matrix in (1), are uncertain. Sengupta [17], later expanded by Wang et al. [18], introduced the concept of Fuzzy DEA as a method to handle data uncertainty. Similar to this approach, Kabnurkar [19] extended and applied the concepts of Fuzzy DEA to incorporate preference bounds when the restriction bounds are uncertain in the direct restriction and assurance region models. The resulting models are linear programs that have restrictions on the value of the weights. Though these bounds are usually different when the bounds are certain, they ultimately still restrict the feasibility region of the resulting mathematical program. As *g-SWAT* uses a penalty function, the feasibility region is unaffected, thus allowing an optimal solution to possibly slightly not adhere to the managerial preference (this is a function of the penalty factor as we will see in Section 3). Both Fuzzy DEA and classical restriction methods require the decision maker to make absolute changes in the weight restriction bounds. In certain situations, such as the Navy example in this paper, this may lead to confusion and additional computation if the decision maker is interested in performance of

[1] We direct the reader to Estellita Lins et al. [7] for a set of infeasible bounds that may arise with standard weight restriction methods.