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Conflict-Free Coloring of points on a line with respect to a set of intervals

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ABSTRACT

We present approximation algorithms for CF-coloring of points on a line with respect to a given set of intervals. For the restricted case where no two intervals have a common right endpoint, we present a 2-approximation algorithm, and, for the general case where intervals may share a right endpoint, we present a 4-approximation algorithm. The running time of both algorithms is $O(n \log n)$.

1. Introduction

The model of *Conflict-Free Coloring* (CF-coloring) was introduced in [6] and further studied in [1,7,8]. This model arises from frequency assignment problems in cellular networks. In such networks each base station is assigned a certain frequency and transmits data in this frequency within some given region. Clients can connect to each other only via base stations and are able to scan frequencies in search for a base station that is received well. The quality of the reception depends on the noise caused by signals transmitted by other base stations that reach the client. In this context, a base station is received well if all other base stations that reach the client are assigned other frequencies and thus cannot interfere.

In the original CF-coloring model, the network is required to serve clients at any location that is reached by some base station. Thus, one must assign frequencies to the base stations, such that every point in each transmission region is supplied. The problem objective is to minimize the number of frequencies assigned.

Note that the CF-coloring problem is essentially different from the regular vertex coloring problem in the corresponding geometric intersection graph, as the latter completely forbids using the same color for intersecting regions. However, the NP-completeness proof of [5] for minimum coloring of unit disks, can be adapted to the case of CF-coloring of unit disks (that is, where each transmission region is a unit disk). Since this proof uses a reduction from coloring planar graphs, it also follows that CF-coloring of unit disks is hard to approximate within ratio $4/3 - \epsilon$, for any $\epsilon > 0$.

Algorithms that use $O(\log n)$ colors (where n is the number of regions) are given in [6] for CF-coloring of disks, axis-parallel rectangles, regular hexagons, and general congruent centrally symmetric convex regions in the plane. It is shown there that for a certain arrangement of the regions, called a "chain", $\Omega(\log n)$ colors are needed in order to supply all the points within the coverage area.

A dual model, in which points are colored with respect to regions was also defined in [6] and further studied in [8]. In this paper we consider a special one-dimensional version of this problem. We are given a set of intervals on the real line, where the left and right endpoints of each interval belong to the set $\{1, 2, \ldots, n\}$. One must assign a color to each of the

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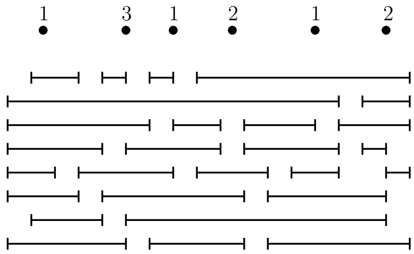

Fig. 1. CF-coloring with respect to a set of intervals; for each interval, at least one of the points that lie in it has a unique color.

points $\{1, 2, \ldots, n\}$, such that all the given intervals are supplied. That is, for each interval there exists a point, among the points lying in the interval, whose color is unique. This problem is equivalent to the problem of CF-coloring a chain of unit disks, where it is necessary to supply only a given subset of the cells of the arrangement of the disks. This idea is actually a natural extension of the original CF-coloring model, since in many applications good reception is needed only in some locations.

Recently, online versions of CF-coloring of points with respect to intervals have been introduced [2,3]. In these versions one must assign colors to points that arrive online, in order to supply all intervals. Note that our model is different since only a given subset of the intervals must be supplied.

In Section 3 we consider two special cases of CF-coloring and present algorithms for these cases, achieving CF-coloring with only two (non-zero) colors. In Section 4 we present a 2-approximation algorithm for CF-coloring of points with respect to a given set of intervals, assuming that no two intervals have a common right endpoint. Later, in Section 5, we remove this assumption and present a 4-approximation algorithm for the general case, where intervals may have common right endpoints. Note that it is still not clear whether this problem is polynomial or not. In Section 6, we analyze the complexity of the algorithms presented in this paper.

2. Preliminaries

Let $\mathcal{P} = \{p_1, p_2, \ldots, p_m\}$ be a set of points on the line, and let $\mathcal{R} = \{I_1, I_2, \ldots, I_n\}$ be a set of intervals. A *CF-coloring of $\mathcal{P}$ with respect to $\mathcal{R}$* is a function $\chi : \mathcal{P} \to \mathbb{N}$, such that for each $I \in \mathcal{R}$, at least one of the points of $\mathcal{P}$ that lie in I has a unique color; we say that such a point p *supports* I and also that its color $\chi(p)$ *supports* I. (Fig. 1 depicts a CF-coloring of a set of points with respect to a set of intervals.) In this paper we use 0 as a null color, that cannot support any interval. A point whose color is 0 is considered colorless. This is also the default color, so that the color of a point whose color was not fixed explicitly is considered to be 0.

By the definition of CF-coloring, we must require that for each interval $I \in \mathcal{R}$ there is a point of $\mathcal{P}$ that lies in I. Also, a point of $\mathcal{P}$ that does not lie in any interval of $\mathcal{R}$ is irrelevant, and can be colored 0. Thus, we assume that each interval of $\mathcal{R}$ covers at least one point of $\mathcal{P}$, and each point of $\mathcal{P}$ lies in at least one interval of $\mathcal{R}$.

For each $I \in \mathcal{R}$, we denote by $l(I)$ and $r(I)$ the left and right endpoints of I, respectively. We denote by $\vdash_I$ (resp. $\dashv_I$) the subset of intervals whose left (resp. right) endpoint is in I (including I itself). That is $\vdash_I = \{I' \colon l(I') \in I\}$ and $\dashv_I = \{I' \colon r(I') \in I\}$.

Similarly, for a point $p \in \mathcal{P}$, we denote by $\dashv_p$ the subset of intervals of $\mathcal{R}$ whose right endpoint is p; that is, $\dashv_p = \{I \in R \colon r(I) = p\}$.

Let $I \in \mathcal{R}$ and let $p_l, p_r \in \mathcal{P}$ be the leftmost and rightmost points of $\mathcal{P}$ that lie in I. The interval $[p_l, p_r]$ is obtained from I by removing from both its ends pieces that are empty of points of $\mathcal{P}$. Obviously, for each point $p \in \mathcal{P}$ it holds that $p \in I$ if and only if $p \in [p_l, p_r]$, and hence we may assume that all intervals $I \in \mathcal{R}$ have their endpoints in $\mathcal{P}$.

Finally, for a subset $\mathcal{R}'$ of intervals, we denote by *Range*$(\mathcal{R}')$ the range defined by the intervals of $\mathcal{R}'$; that is *Range*$(\mathcal{R}') = [\min_{I \in \mathcal{R}'}\{l(I)\}, \max_{I \in \mathcal{R}'}\{r(I)\}]$.

3. Two special cases

In this section we consider two special cases of our CF-coloring problem, in which the set of intervals $\mathcal{R}$ satisfies some additional property. In both cases we obtain a CF-coloring using only two non-zero colors.

3.1. CF-coloring of points with respect to a set of non-nested intervals

Let $\mathcal{P}$ and $\mathcal{R}$ be as above. In addition, assume that there are no two intervals $I_1, I_2 \in \mathcal{R}$ such that $I_1 \subset I_2$. Algorithm NNCFCP (Non-Nested CF-Coloring of Points) outputs a CF-coloring χ of $\mathcal{P}$ with respect to $\mathcal{R}$ using only two non-zero colors.

Clearly, each $I \in IS$ is supported (twice). Assume there is an interval $I \notin IS$ that is not supported. If neither 1 nor 2 occur in I, then, since I is not contained in any other interval of $\mathcal{R}$, we have that $IS \cup \{I\}$ is independent, contradicting the fact that IS is maximum independent. Otherwise, both 1 and 2 occur in I at least twice each, implying that there exists an interval $I' \in IS$ such that $I' \subset I$ — a contradiction. Thus we conclude that χ is a CF-coloring and obtain the following lemma.