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Bichromatic 2-center of pairs of points ☆

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ABSTRACT

We study a class of geometric optimization problems closely related to the 2-center problem: Given a set S of n pairs of points in the plane, for every pair, we want to assign red color to a point of the pair and blue color to the other point in order to optimize the radii of the minimum enclosing ball of the red points and the minimum enclosing ball of the blue points. In particular, we consider the problems of minimizing the maximum and minimizing the sum of the two radii of the minimum enclosing balls. For each case, minmax and minsum, we consider distances measured in the L_2 and in the L_∞ metrics.

1. Introduction

In this paper we consider the following geometric optimization problem:

The 2-Center Color Assignment problem: Given a set S of n pairs of points in the plane, for each pair of S choose one point to be red and the other to be blue, in such a way that a function of the size of the minimum enclosing balls of the set of red points R and the set of blue points B is minimized.

We consider two optimization criteria: the first one is to minimize the maximum of the radii of the minimum enclosing balls of R and B, respectively, while the second one is to minimize their sum. For each criterion, we study the problem for both the L_∞ and the L_2 metrics. Thus, we consider four variants of the 2-Center Color Assignment problem that will be referred to as: the MinMax-L_∞ problem, the MinMax-L_2 problem, the MinSum-L_∞ problem, and the MinSum-L_2 problem.

A natural variant of these problems is the Pairs of Points 1-Center problem, in which the goal is to determine a minimum-radius ball that encloses at least one point of each of the pairs. We consider the corresponding versions of

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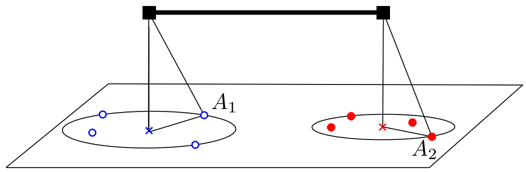

Fig. 1. Schematic of a flow corridor (bold segment) servicing air traffic between blue and red points (airports). The maximum distances between the airports of each color and their closest endpoint depend on the radii of the disks. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

this problem in the L_∞ and L_2 metrics. We refer to them as the Pairs of Points L_∞ 1-Center problem and the Pairs of Points L_2 1-Center problem, respectively.

Motivation In addition to being a natural variant of the fundamental 2-center problem from facility location, our problem is motivated by a problem in "chromatic clustering", the Chromatic Cone Clustering problem, which arises in certain applications in biology, as studied by Ding and Xu [19] and described in the related work section below. We also mention below connections between our model and certain problems in the study of imprecise (or "indecisive") points.

Another motivating view of our problem comes from a transportation problem in which there are origin/destination pairs of points between which traffic flows. We have the option to establish a special high-priority traffic corridor, modeled as a straight segment, which traffic flow is required to utilize in going between pairs of points. The corridor offers substantial benefit in terms of safety and speed. Our goal is to locate the corridor in such a way that we minimize off-corridor travel when traffic between origin/destination pairs utilizes the corridor. Models dealing with alternative transportation systems have been suggested in location theory [7], and simplified mathematical models have been widely studied in order to investigate basic geometric properties of urban transportation systems [2]. Recently, there has been an interest in facility location problems derived from urban modeling. In many cases one is interested in locating a highway that optimizes some given function that depends on the distance between elements of a given point set [5,12,18,28]. Specifically, in this work, we are motivated by an application in air traffic management, in which the use of "flow corridors" (or "tubes") has had particular interest. Flow corridors have been proposed as a potential means of addressing high demand routes by establishing dedicated portions of airspace designed for self-separating aircraft, requiring very little controller oversight [33,34,36,35]. Given a set S of *pairs* of points (origin/destination pairs) in the plane, we want to find two "centers", which define the endpoints of a corridor. Traffic travels from its origin to one endpoint of the corridor, follows the corridor to the other endpoint, then proceeds directly to its corresponding destination; see Fig. 1. Of course, the real air-traffic problem has to take into consideration many other issues, including traffic congestion, sector geometry, fuel consumption, flight dynamics, time, etc. Here, we consider a simplified model in which we assign each airport to one of the two endpoints of the flow corridor and must determine an optimal location for the flow corridor, with the objective of minimizing the distances from airports to their assigned endpoints. In this model, we assume that every pair must utilize the corridor, explicitly ruling out the possibility of going directly between the pair of points; such direct flights may result in unwanted traffic congestion outside the neighborhoods of the corridor endpoints.

Related work Both the 1-center (also called Minimum Enclosing Disk) and the 2-center problem have been widely studied for points in the plane. The 1-center problem for n points in the plane is well known to be solvable in $O(n)$ time using techniques related to linear-programming and prune-and-search [15,21,31]. The 2-center problem has received much attention in recent years; the current best known deterministic algorithm is due to Chan [14], and the current best randomized algorithm is due to Eppstein [22]. The Rectilinear 2-center problem, in which the metric used is L_1 or L_∞, can be solved in linear time [20]. The discrete version was considered by Bespamyatnikh and Segal [9]. However, the restriction on the coloring of the pairs of points that we have in this paper makes our problems rather different, and it seems that we cannot directly apply any similar methods to our case.

Our problem is also similar to the facility location problems with the objective of minimizing the maximum cost of the customers, where the cost of a customer is the minimum between the cost of using the facility and the cost of not using the facility [11]. However, the objective functions we use are more complex, leading to considerably more involved problems.

A problem closely related to ours is the Chromatic Cone Clustering problem of Ding and Xu [19]: Given a point set $\mathcal{G} = G_1 \cup G_2 \cup \ldots \cup G_n \subset \mathbb{R}^d$ formed by the n point sets $G_1, G_2, \ldots, G_n$, such that each G_j consists of k points of positive coordinates (i.e., points in the positive orthant), find k cones $C_1, C_2, \ldots, C_k$ with apex at the origin such that each C_i contains a distinct point from every point set G_j and the total amplitude of the cones is minimized. The authors give a $(1+\varepsilon)$-approximation algorithm for this problem by projecting the points of $\mathcal{G}$ into the unit sphere and finding spheres $C'_1, C'_2, \ldots, C'_k$ of minimum total radius such that each contains a distinct point from every G_j. Observe that for $k = 2$ the input is precisely a set of pairs of points, and each of the two output spheres (defining the cones) contains a different point from each pair. Then the Chromatic Cone Clustering problem for $k = 2$ is directly related to the MinSum-L_2 problem. The running time of Ding and Xu's algorithm, stated for high dimensions d, has the parameter $1/\varepsilon$ as an exponent. In comparison, our problem, formulated in the two-dimensional plane, is solved deterministically and the approximation algorithms have running times polynomial in both n and $1/\varepsilon$.