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Solving the natural wireless localization problem to optimality efficiently ☆

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ABSTRACT

Considered a variation of the art gallery problem, the wireless localization problem deals with the placement of the smallest number of broadcasting antennas required to satisfy some property within a given polygon. The case dealt with here consists of antennas that propagate a unique key within a certain antenna-specific angle of broadcast, so that the set of keys received at any given point is sufficient to determine whether that point is inside or outside the polygon. To ascertain this localization property, a Boolean formula must be produced along with the placement of the antennas.

In this paper, we present an exact algorithm based on integer linear programming for solving the $\mathbb{NP}$-hard natural wireless localization problem. The efficiency of our algorithm is certified by experimental results comprising the solutions of 720 instances, including polygon with holes, of up to 1000 vertices, in less than fifteen minutes on a standard desktop computer.

1. Introduction

The Art Gallery Problem (AGP) [1–3] is a long-standing research topic in Computational Geometry. New problems of this type arose upon the introduction of a novel concept of visibility in which guards are able to see through the gallery boundary [4]. The motivation for this formulation originated from applications to wireless networks, where signals from antennas are not blocked by walls.

To illustrate this situation consider the following folkloric example, which captures the essence of the problem [5]. The owner of a café would like to provide wireless internet access to her customers while preventing those outside her shop to access the network infrastructure. To accomplish this, antennas may be installed, each of which broadcasting a unique (secret) key within an arbitrary but fixed angular range. The goal is to place these antennas and to adjust their angles of broadcast so that customers within the area of the café could be distinguished from those outside simply by having them name the keys received at their location. In a more formal way, one seeks to characterize the polygon corresponding to the area of the shop by means of a monotone Boolean formula whose variables are the keys transmitted by the antennas. Since installation and maintenance of the antennas carry a cost, a natural optimization problem amounts to finding a solution with the minimum number of such devices.

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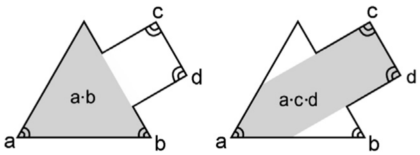

Fig. 1. Polygon with guards on vertices a, b, c and d and Boolean formula $a \cdot b + a \cdot c \cdot d$.

Similarities between this problem and the traditional art gallery problem are self-evident, e.g., guards of the latter correspond to antennas in the former. Notwithstanding that the notions of visibility differ, henceforth we will use the term guard and antenna indistinctly.

As in the classical AGP, the *wireless localization problem* (WLP) has several variants depending on the choice of potential locations for guards, their angular range and maximum visibility distance. In this paper, we assume visibility to be unbounded.

Now, assume that the gallery floor plan is described by a simple polygon P. In the most general situation, guards may be placed anywhere inside P and can broadcast in any direction, in which case they are called *internal guards*. In a more restricted version, guard placement is limited to the vertices of P, and they are referred to as *vertex guards*. Moreover, another situation often found in the literature is the one known as *natural guarding*. Here, the guards are limited to lie on vertices or edges of P and to transmit their signals within the range corresponding to the interior angle of the polygon at that point.

The corresponding *Natural Wireless Localization Problem* (NWLP) is known to be $\mathbb{NP}$-hard [6].

In [5] an alternative $\mathbb{NP}$-hardness proof is given, which can be extended to more general types of guards, such as vertex and internal guards. There are also results [4,7,5] that lead to upper bounds on the number of guards sufficient for coverage, but these bounds are not always tight.

For a polygon with n non-colinear edges, a lower bound of $\lceil n/2 \rceil$ is easy to prove. In [8], it is shown that any *orthogonal* polygon has an optimal solution comprised of $\lceil n/2 \rceil$ guards whose placements can be determined in polynomial time.

To the best of our knowledge, no exact algorithm has been proposed to this date to solve the general NWLP. Furthermore, we are also unaware of any computational experiments reported in the literature for this problem.

Contribution This paper aims at filling these two gaps. To this end, in Sections 3 to 6, we model the NWLP problem as an integer program and in Section 7 we describe ingenious ways to use this formulation algorithmically. Computational results are presented in Section 8 validating this technique as a viable method for computing optimal solutions for instances that include polygons with holes, of up to 1000 vertices. Conclusions and future directions follow.

2. Problem definition and terminology

A guard can be viewed as a wireless station positioned at a given location, which broadcasts a signal in a predefined *angle* and *direction*. The region, $Vis(g)$, covered by a guard g positioned at a point p is the cone with apex at p defined by two rays emanating from it. The bounding rays establish the *angle* and the *direction* of transmission of the corresponding guard in a natural way. Hence, from this point on, a *guard* will be identified to its cone of broadcast: the position of its apex and its angle of transmission.

We may now associate to a guard g a Boolean variable that, for every point p in the plane, takes a true value if and only if p belongs to $Vis(g)$. Given a polygon P and set of guards G, one may ask whether there exists a Boolean formula B on these variables that is satisfied uniquely on the points in P. In the affirmative case, G is said to form a *guarding* of P. Fig. 1 illustrates this idea. For simplicity, in the remainder of the text, Boolean formulas are assumed to be in disjunctive normal form.

In the context of WLP, one is given a guard candidate set G known to contain a guarding of P. When a unitary cost is assigned to each guard in a guarding subset of G, the optimization problem seeks a guarding subset with minimum total cost. Variants of the problem depending on how the set of candidate guards G is defined can be formulated. Usually, G consist of a predefined finite set of locations, broadcasting angles and directions. Common locations for guards are the vertices and edges of P. In this work, we focus on the so-called *natural* guardings and on the resulting optimization problem, NWLP. A guard placed on a vertex of the polygon P is a *natural vertex guard* if its angle is the interior angle at that vertex, relative to P. A guard placed anywhere on an edge of P and broadcasting within an angle of π directed to the interior of P is called a *natural edge guard*. Since any two of these guards on a single edge would cover the same region, we can restrict the placement of natural edge guards to midpoints of edges. Accordingly, we will refer to a guarding consisting only of natural vertex and edge guards as a *natural guarding* [4].

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