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Multi-period efficiency measurement in data envelopment analysis: The case of Taiwanese commercial banks

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ABSTRACT

In measuring the overall efficiency of a set of decision making units (DMUs) in a time span covering multiple periods, the conventional approach is to use the aggregate data of the multiple periods via a data envelopment analysis (DEA) technique, ignoring the specific situation of each period. This paper proposes using a relational network model to take the operations of individual periods into account in measuring efficiencies. The overall and period efficiencies of a DMU can be calculated at the same time. Notably, the overall efficiency is a weighted average of the period efficiencies, and the weights are the most favorable ones for the DMU being evaluated. This model, together with two existing ones, is applied to measure the efficiency of 22 Taiwanese commercial banks for the period of 2009–2011. The three-year multi-period analysis shows that the proposed model is more discriminative than the existing ones in ranking the performance of the banks. The period efficiencies for the three years increased steadily, indicating that the performances of the Taiwanese banks examined in this work were improving over this period.

1. Introduction

Charnes et al. [1] developed the concept of data envelopment analysis (DEA), which has since been widely discussed from both methodological and practical perspectives, and used to measure the relative efficiency of a set of decision making units (DMUs) that use multiple inputs to produce multiple outputs in a specified period of time (see, for example, the review of Cook and Seiford [2] and Seiford [3]). When this approach was initially proposed, it was used for not-for-profit organizations where the input and output factors do not have market values. Later, it was found that it is also applicable to profit-driven companies such as banks [4–7], manufacturing companies [8], hospitals [9], and retail stores [10]. DEA is now a standard technique for performance measurement.

For cases in which the period of time being examined is composed of clearly defined time units, such as years, the total inputs consumed and total outputs produced in all of the periods are aggregated for efficiency measurement. For example, Kao and Hwang [11] used two-year totals to calculate the efficiency of non-life insurance companies in Taiwan. More often, the average inputs and outputs of each period are used. In measuring the performance of Portuguese secondary schools, the inputs used by Portela et al. [12] were the average results of basic education exams in 2005 and 2006. Since DEA has a unit-invariant property [13], the efficiencies calculated from these two types of data, total and average, are the same. When the aggregate data over all the period is used, the resulting efficiency is an overall measure of the performance of the specified period of time, and the specific efficiency of individual periods remains unknown. In this case, the result that a DMU is overall efficient does not necessarily imply that every period is efficient. In fact, it is possible that one period is abnormally inefficient while it is overall efficient, and the abnormal performance may sometimes provide clues about the likelihood of certain events, such as bankruptcy. Therefore, it would be helpful if the period-specific efficiencies could also be known.

If the efficiency of a specific period is desired, it must be calculated separately. However, the efficiencies thus calculated are not comparable among different periods, because the peer groups used for calculating the efficiency in each period are different. One way to solve this problem is window analysis [14], which uses a window of periods to calculate the efficiencies of each DMU in those periods. By considering separate windows, the trend and stability of the performance of each DMU are revealed. However, how to aggregate the period efficiencies of each DMU into the overall efficiency is still a problem.

In response to the weakness of the conventional average-data model, Park and Park [15] developed a model which takes the operations of individual periods into account in measuring the overall efficiency in multiple periods of time. This model is essentially the network DEA model proposed by Färe and Grosskopf [16]

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for measuring the efficiency of a network production system composed of a number of processes connected in parallel operating independently. Although the overall efficiency can be calculated, the efficiency of individual periods must be calculated separately.

The aim of this paper is to develop a model, based on the network DEA approach, to measure the overall efficiency of a set of DMUs in a period of time, taking into account the operations in each period. This model is able to measure the overall and period-specific efficiencies at the same time, and a relationship in which the former is a weighted average of the latter is also derived. It is interesting to note that this model is exactly the same as the one proposed by Kao [17] for measuring the efficiency of a parallel system composed of a number of processes operating independently and applied to measure the teaching and research efficiencies of universities [18].

This paper is organized as follows. In the next section, three models that are able to calculate the overall efficiency of the multi-period system are introduced. The efficiencies of 22 commercial banks in Taiwan in the years 2009, 2010, and 2011 are then evaluated using these models. Based on the period-specific efficiencies, efficiency changes between two periods are evaluated via a global Malmquist productivity index. Finally, a discussion of the results and the conclusions of this work are presented.

2. Multi-period efficiency measurement models

Let X_{ij} and Y_{rj} denote the ith input, $i=1, \ldots, m$, and rth output, $r=1, \ldots, s$, of the jth DMU, $j=1, \ldots, n$, respectively. The CCR model proposed by Charnes et al. [1], under the assumption of constant returns to scale to measure the efficiency of DMU k, can be formulated as follows:

$$
\begin{aligned}
E_k^{CCR} = \max . \ & \sum_{r=1}^{s} u_r Y_{rk} \\
\text{s.t.} \ & \sum_{i=1}^{m} v_i X_{ik} = 1 \\
& \sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0, \quad j=1,\ldots,n \\
& u_r, v_i \ge \varepsilon, \quad r=1,\ldots,s, \ i=1,\ldots,m,
\end{aligned} \tag{1}
$$

where u_r and v_i are virtual multipliers and ε is a small non-Archimedean number used to avoid ignoring any factor in calculating efficiency [19]. Since the efficiency is calculated under the most favorable conditions of the DMU being evaluated, the results are persuasive and acceptable by all DMUs.

Consider a multi-period system composed of q periods, as shown in Fig. 1, where the superscript p in $X_{ij}^{(p)}$ and $Y_{rj}^{(p)}$ denotes the corresponding period. The total quantities of the ith input and the rth output in all q periods for DMU j are $X_{ij} = \sum_{p=1}^{q} X_{ij}^{(p)}$ and $Y_{rj} = \sum_{p=1}^{q} Y_{rj}^{(p)}$, respectively. Several models for measuring the multi-period efficiency of a DMU have been proposed in the literature.

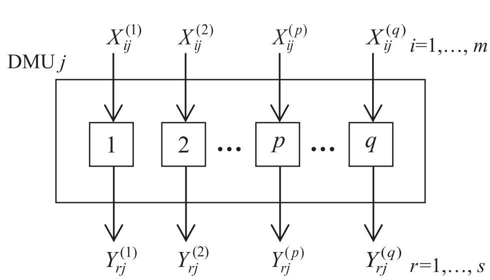

Fig. 1. Structure of the multi-period system.

2.1. Aggregate model

The conventional way of measuring the efficiency of a DMU in a time span covering several periods is to use the average data to represent the general situation, and apply Model (1) to calculate the overall efficiency. As stated previously, due to the unit-invariant property, using the average data of each period (X_{ij}/q, Y_{rj}/q) or the cumulative data of all periods (X_{ij}, Y_{rj}) produces the same result, and this paper uses the latter in its discussion.

Using the total inputs X_{ij} and total outputs Y_{rj} of all of the periods in the time span to measure the overall efficiency of a system via Model (1) implies that the system is treated as a black box, ignoring the operations of individual periods. This model, referred to as aggregate model, has a dual which can be formulated as follows:

$$
\begin{aligned}
E_k^{CCR} = \min . \ & \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^{-} + \sum_{r=1}^{s} s_r^{+}\right) \\
\text{s.t.} \ & \sum_{j=1}^{n} \lambda_j X_{ij} + s_i^{-} = \theta X_{ik}, \quad i=1,\ldots,m \\
& \sum_{j=1}^{n} \lambda_j Y_{rj} - s_r^{+} = Y_{rk}, \quad r=1,\ldots,s \\
& \lambda_j, s_i^{-}, s_r^{+} \ge 0, \quad j=1,\ldots,n, \ i=1,\ldots,m, \ r=1,\ldots,s \\
& \theta \text{ unrestricted in sign.}
\end{aligned} \tag{2}
$$

This shows the production possibility set and a target for DMU k to become efficient.

The aggregate model only calculates the overall efficiency of a DMU in a period of time. If the efficiency of a specific period p is desired, it must be calculated separately by applying the data of that period to Model (1).

2.2. Connected network model

To take the operations of individual periods into consideration in measuring the overall efficiency of q periods, Park and Park [15] developed the following model through extensions of the concept of Debreu–Farrell technical efficiency:

$$
\begin{aligned}
E_k^{PP} = \min . \ & \theta - \varepsilon\left(\sum_{p=1}^{q}\sum_{i=1}^{m} s_i^{-(p)} + \sum_{p=1}^{q}\sum_{r=1}^{s} s_r^{+(p)}\right) \\
\text{s.t.} \ & \sum_{j=1}^{n} \lambda_j^{(p)} X_{ij}^{(p)} + s_i^{-(p)} = \theta X_{ik}^{(p)}, \quad p=1,\ldots,q, \ i=1,\ldots,m \\
& \sum_{j=1}^{n} \lambda_j^{(p)} Y_{rj}^{(p)} - s_r^{+(p)} = Y_{rk}^{(p)}, \quad p=1,\ldots,q, \ r=1,\ldots,s \\
& \lambda_j^{(p)}, \ s_i^{-(p)}, \ s_r^{+(p)} \ge 0, \quad p=1,\ldots,q, \ j=1,\ldots,n, \\
& \quad i=1,\ldots,m, \ r=1,\ldots,s \\
& \theta \text{ unrestricted in sign.}
\end{aligned} \tag{3}
$$

Note that two modifications have been made to the original model, one is that the output orientation form is changed to the input orientation one to be comparable with the aggregate Model (2). The other is that the convexity constraints of $\sum_{j=1}^{n} \lambda_j^{(p)} = 1, \ p=1,\ldots,q$, for variable returns to scale are removed to conform to the constant returns to scale of Model (2).

A closer examination of Model (3) shows that it is exactly the same as the one formulated from the network DEA model of Färe and Grosskopf [16] for the multi-period structure of Fig. 1. This model treats the q periods as independent processes in formulating the constraints, in that each period p has its own set of intensity coefficients, $\lambda_j^{(p)}, j=1, \ldots, n$; only the distance measure, θ, is the same for all periods. Since the q periods are connected via θ, it is thus called a connected network model.

When the superscript p in Model (3) is fixed at a specific value t, that is, only the data set of period t of all n DMUs is used,