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Design of solids for antigravity motion illusion

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A R T I C L E I N F O

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A B S T R A C T

This paper presents a method for designing solid shapes containing slopes where orientation appears opposite to the actual orientation when observed from a unique vantage viewpoint. The resulting solids generate a new type of visual illusion, which we call “impossible motion”, in which balls placed on the slopes appear to roll uphill thereby defying the law of gravity. This is possible because a single retinal image lacks depth information and human visual perception tries to interpret images as the most familiar shape even though there are infinitely many possible interpretations. We specify the set of all possible solids represented by a single picture as the solution set of a system of equations and inequalities, and then relax the constraints in such a way that the antigravity slopes can be reconstructed. We present this design procedure with examples.

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1. Introduction

This paper presents a computational approach to design a new visual illusion. Visual illusion is a perceptual behavior where what we “see” differs from the physical reality. This phenomenon is important in vision science because it helps us to understand the nature of human perception [7,8]. Numerous traditional visual illusions are known, most of which are generated by two-dimensional pictures and their motions [5,15].

However, very few visual illusions are known that make use of three-dimensional solid shapes. An early example was the Ames room, where a person looks taller when he moves from one corner of the room to another [3]. Other examples include impossible solids produced by a hidden-gap trick [1], and those without hidden gaps [13]. The latter class was extended to include a new type of illusion called “impossible motion” [14].

The design of illusions using solids requires mathematics, because this process can be counterintuitive.

This paper concentrates on one class of such solids called “antigravity slopes”, in which balls appear to roll uphill against the law of gravity and produce appearances of an “impossible motion”.

In Section 2, we briefly review picture interpretation theory, which specifies the set of all possible solids represented by a picture, and in Section 3 we show that antigravity slopes cannot be constructed using that formulation. In Section 4, we remove some of the constraints by changing structures in the hidden part so that design of antigravity slopes becomes possible. We show some examples in Section 5, and provide concluding remarks in Section 6.

2. Reconstruction of a solid from a picture

Our goal is to construct solids that generate a visual illusion. As a tool to achieve this goal, let us review picture interpretation theory, by which we can specify all solids represented by a given picture.

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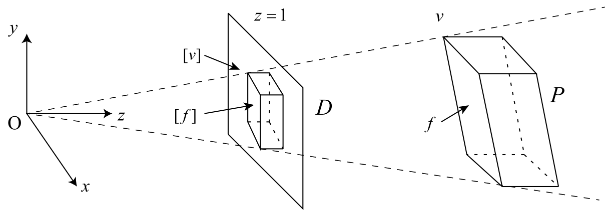

Fig. 1. Solid and its central projection.

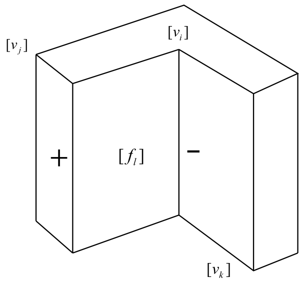

Fig. 2. ON, NEARER and FARTHER predicates between faces and vertices.

For two points p and q, let $\overline{pq}$ denote the closed line segment connecting p and q. Let $(p_1, p_2, \dots, p_n)$ be a sequence of mutually distinct points in two-dimensional space. We assume that the line segments $\overline{p_1 p_2}, \overline{p_2 p_3}, \dots, \overline{p_{n-1} p_n}$ and $\overline{p_n p_1}$ do not intersect except at their terminal points. Then, the region bounded by the cyclic sequence of these line segments is called a *polygon* (not necessarily convex). The points $p_1, p_2, \dots, p_n$ are called *vertices* and the line segments $\overline{p_1 p_2}, \overline{p_2 p_3}, \dots, \overline{p_{n-1} p_n}$ and $\overline{p_n p_1}$ are called *edges* of the polygon. Intuitively a polygon is a piece of hard thin flat plate whose boundary is composed of line segments. We place a finite number of polygons in three space, and thus construct a solid object. Formally we define a *solid object* as a collection of a finite number of polygons placed in three space. A solid object is also called a *solid* for short. The polygons that constitute a solid are called *faces*.

For example, the solid P in Fig. 1 is a hexahedron. This object can be considered a solid composed of the six boundary faces; we do not care whether the inside is occupied with material or empty.

Our goal is to construct an antigravity slope, which is a solid object typically composed of a base plate, slopes, and supporting columns. A slope is composed of a slide and two side walls; they can all be considered as polygons. A support column is a polyhedron, but we consider it as a collection of surface polygons. As shown in Fig. 1, let P be a solid object fixed to three space with the (x, y, z) Cartesian coordinate system, and D be its projection on the plane $z = 1$ with respect to the center of projection at the origin O. This means that we see the object from the viewpoint at the origin, and get its image on the plane $z = 1$. We assume that all the edges of P are drawn in D, and hence D is called the *line drawing* of P. Let f be a face of a solid. We denote by $[f]$ the image of f projected on the plane $z = 1$. Similarly for a vertex v of a solid, we denote by $[v]$ the image of v on the plane $z = 1$.

If P is given, D is uniquely determined. On the other hand, if D is given, an associated P is not unique in general. If D represents a solid object correctly, there are infinitely many solids that generate D. If D is incorrect in the sense that it does not represent a solid object, there is no corresponding P. So we consider how to specify the set of all solid objects that can generate D.

Suppose that we are provided with D and the relative relations among the vertices and the faces of the solid object. Let $V = \{v_1, v_2, \dots, v_n\}$ and $F = \{f_1, f_2, \dots, f_m\}$ be the set of vertices and of faces, respectively, of a solid in three space. Let $\mathrm{ON}(v_i, f_j)$ represent the predicate stating that the vertex v_i is on the face f_j. Similarly let $\mathrm{NEARER}(v_i, f_j)$ represent "v_i is nearer than the plane containing f_j", and $\mathrm{FARTHER}(v_i, f_j)$ represent "v_i is farther than the plane containing f_j", where "near" and "far" are meant according to the distance from the viewpoint at the origin to each part of the object.

For example, consider the line drawing in Fig. 2. Let us concentrate on the three vertices v_i, v_j, v_k and face f_ℓ in this figure. Since vertex v_i is on face f_ℓ, we get

$$\mathrm{ON}(v_i, f_\ell).$$

The edge labeled $+$ in Fig. 2 represents a ridge of a mountain, and hence, if we extend the plane f_ℓ, it passes between the viewpoint and the vertex v_j. Hence we get

$$\mathrm{FARTHER}(v_j, f_\ell).$$

The edge labeled $-$, on the other hand, forms the bottom of a valley, and hence the face f_ℓ when extended goes beyond v_k. Hence we get

$$\mathrm{NEARER}(v_k, f_\ell).$$

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