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Hybrid censoring: Models, inferential results and applications

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ABSTRACT

A hybrid censoring scheme is a mixture of Type-I and Type-II censoring schemes. In this review, we first discuss Type-I and Type-II hybrid censoring schemes and associated inferential issues. Next, we present details on developments regarding generalized hybrid censoring and unified hybrid censoring schemes that have been introduced in the literature. Hybrid censoring schemes have been adopted in competing risks set-up and in step-stress modeling and these results are outlined next. Recently, two new censoring schemes, viz., progressive hybrid censoring and adaptive progressive censoring schemes have been introduced in the literature. We discuss these censoring schemes and describe inferential methods based on them, and point out their advantages and disadvantages. Determining an optimal hybrid censoring scheme is an important design problem, and we shed some light on this issue as well. Finally, we present some examples to illustrate some of the results described here. Throughout the article, we mention some open problems and suggest some possible future work for the benefit of readers interested in this area of research.

1. Introduction

Type-I and Type-II censoring schemes are the two most common and popular censoring schemes. In Type-I censoring scheme, the experimental time is fixed, but the number of observed failures is a random variable. On the other hand, in Type-II censoring scheme, number of observed failures is fixed, but the experimental time is a random variable. The mixture of Type-I and Type-II censoring schemes is known as hybrid censoring scheme, and it can be described as follows.

Consider the following life-testing experiment in which n units are placed on test. The lifetimes of the sample units are assumed to be independent and identically distributed (i.i.d.) random variables. Let the ordered lifetimes of these units be denoted by $X_{1:n}, \ldots, X_{n:n}$, respectively. The test is terminated when a pre-fixed number, $r < n$, out of n items has failed, or when a pre-fixed time, T, has been reached. In other words, the life-test is terminated at a random time $T_* = \min\{X_{r:n}, T\}$. It is also usually assumed that the failed units are not replaced in the experiment.

This hybrid censoring scheme, which was originally introduced by Epstein (1954), has been used quite extensively in reliability acceptance test in MIL-STD-781-C (1977). From now on, we refer to this hybrid censoring scheme as *Type-I hybrid censoring scheme* (*Type-I HCS*). It is evident that the complete sample situation as well as Type-I and Type-II right censoring schemes are all special cases of this Type-I HCS.

Since the introduction of Type-I HCS by Epstein (1954), extensive work has been done on hybrid censoring and many different variations of it. In his pioneering work, Epstein (1954) introduced the Type-I HCS and considered the special case when the lifetime distribution is exponential with mean lifetime θ. He discussed estimation methods for θ and also proposed

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a two-sided confidence interval for θ, without presenting a formal proof of its construction. Later, Fairbanks et al. (1982) modified slightly the proposition of Epstein (1954), and suggested a simple set of confidence intervals. Motivated by the works of Bartholomew (1963), and Barlow et al. (1968), Chen and Bhattacharyya (1988) derived the exact distribution of the conditional maximum likelihood estimator (MLE) of θ by using the conditional moment generating function approach, and used it to construct an exact lower confidence bound for θ. Childs et al. (2003) derived a simplified but an equivalent form of the exact distribution of the MLE of θ as derived by Chen and Bhattacharyya (1988). In constructing exact confidence intervals for θ from the exact conditional densities, these authors made a critical assumption about monotonicity of tail probabilities and this was formally proved only recently by Balakrishnan and Iliopoulos (2009). Draper and Guttman (1987) considered the Bayesian inference for θ, and obtained the Bayesian estimate and a two-sided credible interval for θ by using an inverted gamma prior. A comparison of different methods of estimation, by using extensive Monte Carlo simulations, was carried out by Gupta and Kundu (1998). While all these results were developed for the case of the exponential distribution, Type-I HCS has been discussed for some other lifetime distributions such as two-parameter exponential, Weibull, log-normal and generalized exponential. All these developments pertaining to this Type-I HCS will be reviewed in Section 2.

As in the case of conventional Type-I censoring, the main disadvantage of Type-I HCS is that most of the inferential results need to be developed in this case under the condition that the number of observed failures is at least one; moreover, there may be very few failures occurring up to the pre-fixed time T which results in the estimator(s) of the model parameter(s) having low efficiency. For this reason, Childs et al. (2003) introduced an alternative hybrid censoring scheme that would terminate the experiment at the random time $T^* = \max\{X_{r,n}, T\}$. This hybrid censoring scheme is called *Type-II hybrid censoring scheme* (*Type-II HCS*), and it has the advantage of guaranteeing at least r failures to be observed by the end of the experiment. If r failures occur before time T, then the experiment would continue up to time T which may end up yielding possibly more than r failures in the data. On the other hand, if the r-th failure does not occur before time T, then the experiment would continue until the time when the r-th failure occurs in which case we would observe exactly r failures in the data. All developments pertaining to this Type-II HCS will be reviewed in Section 3.

In a direct comparison of Type-I and Type-II hybrid censoring schemes, we observe the following advantages and disadvantages in them:

Type-I HCS: In this case, the termination time is pre-fixed, which is clearly an advantage from an experimenter's point of view. However, if the mean lifetime of experimental units is somewhat larger than the pre-fixed time T, then with high probability, far fewer failures than the pre-fixed r would be observed before time T. This will definitely have an adverse effect on the efficiency of inferential procedures based on Type-I HCS.

Type-II HCS: In this case, the termination time is a random variable, which is clearly a disadvantage from the experimenter's point of view. On the other hand, at least r failures and possibly more than r failures would be observed by the termination time, and this will result in efficient inferential procedures in this case.

To overcome some of the drawbacks in these schemes, Chandrasekar et al. (2004) introduced two extensions, and called them *generalized Type-I and Type-II hybrid censoring schemes*. We will review in Section 4 all developments relating to these generalized Type-I HCS and Type-II HCS. Recently, Balakrishnan et al. (2008) combined all these censoring schemes, and introduced the so-called *unified hybrid censoring scheme*. Details on this unified hybrid censoring scheme and related inferential issues will be reviewed in Section 5.

In many life-testing experiments, quite often there may be more than one risk factor that could cause the failure of units. In such situations, an investigator would naturally be interested in the assessment of a specific risk factor in the presence of all other risk factors. Such a scenario is referred to in the literature as the *competing risks* problem. Analysis of data from hybrid life-testing experiments in the presence of competing risks was discussed by Kundu and Gupta (2007), and this will be the focus of discussion in Section 6.

Recently, *progressive censoring* has been discussed quite extensively in the literature as a very flexible censoring scheme since it allows the removal of live experimental units at various intermittent times during the experiment in addition to removal at the termination of the experiment. For an elaborate treatment on the theory, methods and applications of progressive censoring, one may refer to the monograph of Balakrishnan and Aggarwala (2000) and the recent review article by Balakrishnan (2007). Hybrid censoring schemes have been introduced in the context of progressive censoring as well. Inferential results have been developed by Kundu and Joarder (2006) and Childs et al. (2008) based on such *progressive hybrid censoring schemes*, and these results will be reviewed in Section 7.

Ng et al. (2009) and Lin et al. (2009) have proposed *adaptive progressive hybrid censoring schemes* and then developed inferential methods for the unknown parameter(s) of exponential and Weibull distributions, respectively. Such adaptive progressive hybrid censoring schemes allow the experimenter to modify the censoring scheme adaptively during the life-testing experiment. The corresponding models and inferential results will be reviewed in Section 8.

Often, in reliability and life-testing experiments, one is interested in the effects of varying stress factors, such as temperature, voltage and load, on the lifetimes of experimental units. Since many modern products are highly reliable and hence a life-test under normal condition would tend to last a long time, experimenters often resort to an *accelerated life-testing* (ALT) experiment. Such experiments allow the experimenter to obtain adequate data for the items under accelerated stress conditions, which cause the items to fail much more quickly than under normal operating conditions. For some key references in the area of accelerated testing, one may refer to Nelson (1990), Meeker and Escobar (1998), Bagdonavicius and Nikulin (2002), and the references cited therein. A special class of the ALT is called *step-stress testing*, which allows the