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The bike sharing rebalancing problem: Mathematical formulations and benchmark instances

Mauro Dell'Amico [a], Eleni Hadjicostantinou [b,c], Manuel Iori [a,*], Stefano Novellani [a,d]

[a] DISMI, University of Modena and Reggio Emilia, Via Amendola 2, 42122 Reggio Emilia, Italy
[b] Business School, Imperial College London, South Kensington Campus, London SW7 2AZ, UK
[c] School of Engineering, Frederick University, 7 Y. Frederickou Str., 1036 Nicosia, Cyprus
[d] DEI – "Guglielmo Marconi", University of Bologna, Viale Risorgimento 2, 40136 Bologna, Italy

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ABSTRACT

Bike sharing systems offer a mobility service whereby public bicycles, located at different stations across an urban area, are available for shared use. These systems contribute towards obtaining a more sustainable mobility and decreasing traffic and pollution caused by car transportation. Since the first bike sharing system was installed in Amsterdam in 1965, the number of such applications has increased remarkably so that hundreds of systems are now operating all over the world.

In a bike sharing system, users can take a bicycle from a station, use it to perform a journey and then leave it at a station, not necessarily the same one of departure. This behavior typically leads to a situation in which some stations become full and others are empty. Hence, a balanced system requires the redistribution of bicycles among stations.

In this paper, we address the Bike sharing Rebalancing Problem (BRP), in which a fleet of capacitated vehicles is employed in order to re-distribute the bikes with the objective of minimizing total cost. This can be viewed as a special one-commodity pickup-and-delivery capacitated vehicle routing problem. We present four mixed integer linear programming formulations of this problem. It is worth noting that the proposed formulations include an exponential number of constraints, hence, tailor-made branch-and-cut algorithms are developed in order to solve them.

The mathematical formulations of the BRP were first computationally tested using data obtained for the city of Reggio Emilia, Italy. Our computational study was then extended to include bike sharing systems from other parts of the world. The information derived from the study was used to build a set of benchmark instances for the BRP which we made publicly available on the web. Extensive experimentation of the branch-and-cut algorithms presented in this paper was carried out and an interesting computational comparison of the proposed mathematical formulations is reported. Finally, several insights on the computational difficulty of the problem are highlighted.

1. Introduction

Bike sharing systems offer a mobility service in which public bicycles are available for shared use. These bicycles are located at stations that are displayed across an urban area. The users of the system can take a bicycle from a station, use it for a journey, leave it into a station (not necessarily the one of departure), and then pay according to the time of usage.

These systems are an important instrument used by public administrations to obtain a more sustainable mobility, decrease traffic and pollution caused by car transportation, and solve the so-called last mile problem related to proximity travels. From the first bike sharing system installed in Amsterdam in 1965, their number increased in the following years to reach, in 2011, more than 400 systems only in Europe, see, e.g., DeMaio [1] and project OBIS [2]. In North America the implementation of bike sharing systems started only in 2008, see Pucher et al. [3], but as far as we know it already counts more than 20 operating systems. In the rest of the world the number of systems is rising at a very high rate, as discussed, e.g., by Shaheen et al. [4].

Stations are made of different slots, each of which hosts a single bicycle. In modern systems, stations are connected to the Internet and display in real time the occupation status of each slot. In this way users can easily check where it is possible to pick up or drop a bicycle. The usage of the system is monitored continuously, and the collected information is used to improve the level of service.

Operating bike sharing systems has a cost that may vary greatly (depending on the system itself, the population density, the service area and the fleet size), with a consistent impact on the budget of the public administration. The setup costs for installing the system

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[*] Corresponding author. Tel.: +39 0522 522653.
E-mail addresses: mauro.dellamico@unimore.it (M. Dell'Amico), e.hconstantinou@imperial.ac.uk (E. Hadjicostantinou), manuel.iori@unimore.it (M. Iori), stefano.novellani2@unibo.it (S. Novellani).

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include, among others, the cost of purchasing the bikes, the slots, and the stations, and the cost of the back-end system used to operate the equipment, see, e.g., DeMaio [1]. The daily operating costs include maintenance, insurance, possibly website hosting and electricity, and, most important, the cost due to the redistribution of bikes among the stations. Indeed, at the end of a day some stations are typically full and others are empty.

A commonly adopted rule for rebalancing is to keep each station only partly occupied, i.e., there should always be in a station some slots occupied by bicycles, to allow users to pick them up, and some free slots, to allow users to drop a bicycle at the end of their journey. Let us suppose that a desired level of occupation is present in the early morning in a given bike station, then the number of bikes may change drastically during the day from the desired level because of the users' travel behavior. This happens typically in cities characterized by a hilly territory, see, e.g., Kaltenbrunner et al. [5], where users take a bike from a station located at the top of a hill, leave it at the bottom and then take the journey back with different means of transportation. It is also common for cities located in flat areas, where some stations have large inflows or outflows at different times of the day. In the next section we report the results of the analysis of the system at the city of Reggio Emilia (Italy) used over a period of seven months.

Repositioning is usually done by means of capacitated vehicles based on a central depot that pick up bicycles from stations where the level of occupation is too high and deliver them to stations where the level is too low. Usually a buffer of bicycles is kept at the depot, and used to allow a more flexible redistribution. The resulting optimization problem of deciding how to route the vehicles so as to perform the redistribution at minimum cost is known in the literature as the *Bike sharing Rebalancing Problem* (BRP), and has recently attracted the interest of many researchers and practitioners in the area. It can be modeled as either a *dynamic* or a *static* optimization problem. In the static version, a snapshot of the level of occupation at the stations is taken and then used to plan the redistribution. In the dynamic version, the real-time usage of the system is taken into account, and the redistribution plan is possibly updated as soon as the information required to make decisions is revealed over time.

Usually, static rebalancing is associated with a redistribution process that is performed during the night, when the system is kept closed or the demand is very low, whereas dynamic rebalancing is associated to redistributions operated during the day, when demand may be high. In the real-world case that we studied in detail the redistribution is performed during the night, and hence we focus on the static version of the problem.

In this paper we provide several contributions. In Section 2 we briefly present the real-world case study that we conducted at the city of Reggio Emilia, by analyzing the travel flows, the users behavior and the resulting levels of occupation at the stations. In Section 3 we formally describe the BRP and discuss the related literature. In Section 4 we propose four *Mixed Integer Linear Programming* (MILP) formulations to model the problem. All these formulations involve an exponential number of constraints, so Section 5 presents the branch-and-cut algorithms that we implemented to solve them. We present a large set of benchmark instances in Section 6, obtained by analyzing the usage of several bike systems around the world; we make these instances publicly available on the Internet. Extensive computational results of the branch-and-cut algorithms are reported in Section 7. Finally, conclusions are given in Section 8 and future research directions are discussed.

2. Data analysis of a real-world case

The first real-world case that we studied is the bike sharing system of Reggio Emilia, a city of around 170 thousand inhabitants located in a very flat area in northern Italy. The system, which is depicted in Fig. 1, is quite small and now counts one depot (indicated by 0 in the figure), 13 stations and about 100 bicycles. It operates all day but is kept closed during the night, which is quite common for bike sharing systems in small/medium cities. The redistribution of the bicycles is carried out during the night, by means of a single vehicle that visits each station exactly once.

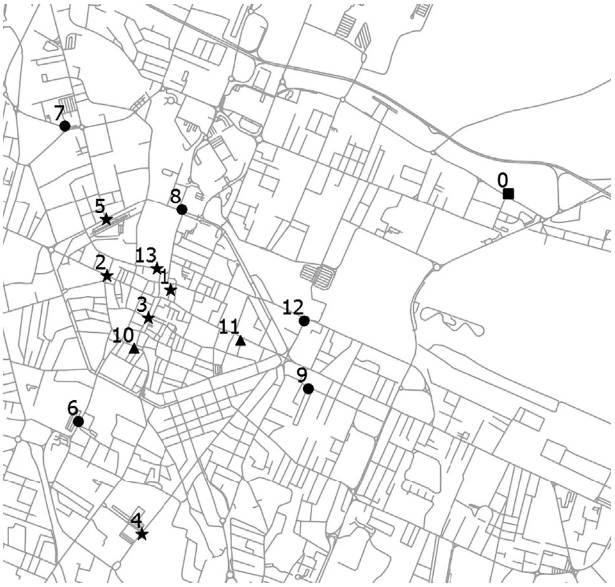

Fig. 1. The bike sharing system of Reggio Emilia. The depot is depicted by 0. Stars, circles and triangles represent stations of the first, second and third groups, respectively.

The data associated with a seven-month usage of the system was provided to us by the municipality of the city. It contains the list of journeys performed by the users in the considered period, including time and station of departure and arrival of each journey. For each station we evaluated the net flow of bicycles on a daily basis, computed as the difference between the inflow and the outflow. This gives the difference between the bicycles available at the beginning of the day and those left at the end of the day in each station. We then plotted the distribution of the net flow over the period, see the distribution graphs in Fig. 2. The x-axis gives the difference Δ between arrivals and departures in a station per day, and the y-axis gives the percentage of times Φ (frequency of occurrence) this number appears throughout the period that we studied. We mostly found normal-like distributions, as the one depicted in Fig. 2(a) for station 4, but also a bimodal one for station 5, see Fig. 2(b). In both cases, for the majority of the days over the observation the stations ended up with a number of bikes different from that available at the beginning of the day, and this supports the choice of performing rebalancing operations.

Furthermore, by analyzing the net flow per hour of all the stations we have been able to determine the diverse variability in usage by the customers. We could consequently divide the stations into three groups, as shown in Fig. 3. In this figure, the x-axis represents the hour τ of the day, and the y-axis states the cumulative number ν of bikes arriving into or departing from the station within a seven-months period. More specifically:

1. The first group, see Fig. 3(a), has a peak of incoming bikes between 7 and 9 am, and a smaller one between 1 and 3 pm. The peaks of outgoing bikes occur between 12 am and 2 pm and between 4 and 6 pm. These stations are all situated in the city center, with the exception of one that is located near the hospital (stations 1, 2, 3, 4, 5 and 13). This usage fits well with