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Deterministic models for premature and postponed replacement

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ABSTRACT

We examine the possibilities of premature and postponed replacement in a deterministic infinite horizon model when there is technological progress. Both revenue and operating cost deteriorate with age, but at different rates. The optimal deterministic replacement time is an implicit solution from the timing boundary obtained for the equivalent real option model using a dynamic programming framework, and then by setting the underlying volatilities equal to zero. A step change improvement characterizing technological progress in the initial operating cost level for the successor occurring during the economic lifetime of the incumbent justifies premature replacement, compared to the traditional present value approach. This finding can be extended to step change improvements in the initial revenue level for the successor and for the re-investment cost. In contrast, if the technological progress can be characterized by a constant declining rate for the initial operating cost level for the successor, then the replacement is postponed for certain parameter values. This finding can be extended to different assumed improvement rates in the initial revenue level for the successor and for the re-investment cost.

1. Introduction

We create within a single formulation an explanation for premature and postponed replacement in the context of an infinite-horizon deterministic model, when there is technological progress. Premature denotes a replacement earlier than justified under traditional capital budgeting (without technological progress), and postponed denotes a later replacement time. The replacement policy is expressed as the implicit solution to a set of analytical relationships representing the timing boundary that are derived from applying a continuous dynamic programming framework. The solution is not expressed dependent on time, but is framed in terms of the operating cost thresholds (given revenue thresholds) that signal an optimal replacement event.

When a replaceable asset is installed, financial managers would normally assess its expected economic lifetime from a standard net present value (NPV) analysis for an infinite replacement chain. This solution, though, is only strictly applicable for like-for-like replacements, but there are many assets with embedded technological progress that violate this assumption, including vehicles and aircraft with higher fuel efficiency, robotic machine tools with greater functionality, mobile phones and computer-based products with faster and novel facilities. The traditional ex-ante economic lifetime (the ex-ante solution from a NPV analysis) will seldom coincide with justifiable replacement time if there is expected or sudden technological progress. Also, since the economic lifetime depends not only on the equipment deterioration rate, but also on the technological progress embedded in the successor due to the incumbent implied obsolescence, the traditional NPV method is likely to be problematic because of its in-built assumption of an equal cycle time. In this paper, we adopt a dynamic programming framework for overcoming the equal cycle time assumption and for establishing analytically the conditions explaining the difference between the traditional ex-ante and justifiable ex-post economic lifetimes.

Although economic lifetime replacement models date back to Faustmann in 1849 on optimal tree stand policy [1], the first application to equipment is apparently made by [2]. This analysis was extended by [3], while [4] simplifies the findings by assuming operating cost behavior to be time dependent. These and subsequent models [5] are predicated on like-for-like replacements in an infinite chain and an equal economic lifetime for every incumbent. However, by building on [6,7], Caplan argues that the ex-ante and ex-post economic lifetimes may differ due to technological progress and establishes that when the change is unforeseen by the incumbent owner while being embodied in the successor, the active economic lifetime is shortened [8]. In contrast, a simulation using a finite-horizon dynamic programming formulation shows that the ex-post exceeds the ex-ante

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economic lifetime for predictable technological progress [9], although this finding is contested by [10]. Practical issues associated with replacement are considered in [11].

The equal-lifetime requirement imposes the repeatability assumption of stationary cost behavior and an infinite horizon. Since the presence of technological progress violates the stationarity assumption, several authors analyze present value infinite-horizon formulations for testing the adequacy of the equal-lifetime hypothesis. Because of the lost opportunity of a foregone improvement, technological progress embodied in the successor may extend the economic lifetime for the incumbent [12,13], and the equal-lifetime assumption can yield in certain circumstances a sub-optimal solution [14,15]. Other authors propose a dynamic programming formulation for discovering numerically the optimal replacement policy,16–19], and in the presence of technological change [20,21]. For non-stationarity, the equal-lifetime rule is shown to yield erroneous results [22–25]. Alternative forms of technological progress are considered in [26,27]. Numerical evaluations potentially impaired owing to a finite-horizon assumption has prompted the search for a finite horizon beyond which the current decision always remains optimal [20,28,29]. Related research seeks a more rational way for terminating the infinite chain [30,31]. When predicting the future is unreliable, some authors advocate the use of a two-cycle replacement model [32–35].

Real projects having managerial flexibility are more appropriately analyzed by applying a real options evaluation [36,37]. Although originally designed for stochastic and not deterministic factors, a real options approach is useful for finding the deterministic replacement policy because of the implicit managerial flexibility in making the replacement decision. In a real option model, the replacement value is conceived as a perpetual American call option [38], which is exercised when the improved value due to the replacement adequately compensates for the re-investment cost as well as the loss of the option value. We apply the dynamic programming method for deriving the timing boundary that discriminates between the replacement and the continuance decisions [36]. The timing boundary for a one-factor model is formulated by [39–41], and for a two-factor model by [42]. The deterministic replacement policy is then derived from the real option timing boundary by setting the volatilities for the stochastic factors equal to zero [36]. The replacement model can be extended to consider the effect of technology exchange [43] and property development [44], while the effect of technological progress on the investment decision is examined in the strategic context [45,46], for complementarity between technologies [47,48] and for technology patronage [49]. Other related work involving real options include revenue maximization [50], R&D decisions in enhancing value [51], and choices over procurement contract terms [52].

Our method is sufficiently versatile for dealing with distinct growth rates for each factor and with model enlargement from increasing the number of factors. There are advantages in adopting this in preference to existing methods for determining the optimal replacement policy in the presence of non-stationarity. Unlike most existing replacement models, revenue is included as a factor, since assets such as cruise ships, vehicle rentals and entertainment facilities experience revenue deterioration with usage, and because it provides a natural way for terminating the chain [2]. By expressing the solution in terms of thresholds, the equal-lifetime requirement is avoided. There is no simplification due to forcing the planning horizon to be finite thereby eliminating any solution impairment arising from omitting residual quantities and goodwill. Finally, this method is less onerous computationally than discrete dynamic programming evaluations.

The paper is organized in the following way. In Section 2, we develop quasi-analytical solutions to the timing boundary for justifying premature and postponed replacement. The behavior of the solution is investigated in Section 3 through numerical illustrations. Section 4 is a conclusion. The dynamic programming framework that is used in determining the timing boundaries is explained in detail in Appendix A, while Appendices B and C provide proofs underpinning some partial derivatives for premature and postponed replacement, respectively.

2. Models of replacement

We consider a durable productive asset, subject to both input and output decay, [53], whose efficiency diminishes progressively and deterministically with time. At any time, the revenue rendered by the asset, denoted by P, changes at a continuous geometric rate θ_P, assumed to be negative, while its operating cost, denoted by C, changes at the continuous geometric rate θ_C, assumed to be positive. For each of these two factors, the rate of deterioration due to usage or passage of time is assumed to remain *constant*. When the incumbent attains a to be determined threshold, it is replaced by a successor at the re-investment cost of K.

We assume that any improvement in the performance due to technological progress is manifested through only one of the three attributes for the successor. These attributes are the initial operating cost level for the successor, its initial revenue level, and its re-investment cost. As an illustration, technological progress embodied in the development of commercial aircraft construction could be in the form of more fuel efficient engines and reduced body weight due to advanced materials. These improvements could result in lower initial operating cost level due to fuel efficiency, in higher revenue level due to the increased payload, and a purchase price change that adapts to the supplier's improved cost structure. Although technological progress can have these manifold effects, we confine our analysis here to a single attribute, the initial operating cost level. It is straightforward to replicate our analysis for the other two attributes. Technological progress is represented differently in each of our three models. In Model I, there is no presumed technological progress. It follows that the attributes for the incumbent and successor have identical levels and this formulation leads to the traditional equal economic lifetime solution. In Model II, there is a step change improvement in the attribute level for the successor that occurs during the economic lifetime for the incumbent, but which is not known at the time of the installation of the incumbent. For this model, we show that the appearance of a successor having an improved attribute level relative to the incumbent results in premature replacement. Finally, Model III represents the case of continuous dynamic but known improvements in the attribute level, which characterizes Moore's Law that the transistor count doubles every two years. Consistent with other analyses, technological progress is formulated as an initial operating cost for the successor that declines geometrically at a known rate, which results in replacement being postponed. Throughout, we assume that technological progress does not influence the rates of deterioration due to usage.

2.1. Model I: no technological progress

Model I characterizes the traditional representation for identifying the optimal cycle time between successive replacements, denoted by $\hat{T}_1$, which is determined from maximizing the value W for an infinite chain of identical assets. The revenue and operating cost levels for the incumbent at installation are denoted by P_0 and