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Estimation in the three-parameter inverse Gaussian distribution

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Abstract

A mixed moments method for the estimation of parameters in the three-parameter inverse Gaussian distribution (IG3) is introduced. The method is an adaptive iterative procedure, which combines the method of moments with a regression method based on the empirical moment generating function. Monte Carlo results indicate that the new procedure is more efficient than alternative estimation methods (including the maximum likelihood) over large portions of the parameter space with samples of small or moderate size. Asymptotic results are also obtained and may be used to draw approximate inferences with small samples. Two data sets are used to illustrate estimation and testing procedures and to construct exploratory graphs for the appropriateness of the IG3 model.

Keywords: Empirical moment generating function; Maximum likelihood estimation; Method of moments; Mixed-moments estimation; Adaptive estimation

1. Introduction

The inverse Gaussian distribution is a well-known competitor of the Weibull, gamma and lognormal distributions in modelling asymmetric data from various scientific fields.

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In reliability and life testing, the inverse Gaussian distribution is particularly useful in situations where early failures dominate. This is due to the non-monotonic behaviour of its hazard function—see e.g. Chhikara and Folks (1989).

In many applications arising in life testing, reliability analysis, meteorology and hydrology it is reasonable to assume that there is a non-zero origin below which no measurement can occur. In the context of life testing, this threshold value is usually interpreted as a guarantee time during which failures cannot occur. Thus, it is useful to consider the three-parameter inverse Gaussian distribution (IG3) model, in which the threshold value is represented by an unknown location parameter.

The IG3 was introduced in the statistical literature by Padgett and Wei (1979). Parameter estimation by the methods of moments and maximum likelihood (ML) was investigated by Padgett and Wei (1979), Cheng and Amin (1981) and Jones and Cheng (1984). Modifications of the moment and ML procedures were considered by Chan et al. (1984), Cohen and Whitten (1985, 1988) and Balakrishnan and Cohen (1991). More recently, Desmond and Yang (1998) considered Bayesian estimation for the location parameter. They demonstrated that the Bayesian approach has superior performance over ML, even when a non-informative prior is used, when the sample is not large. However, no Bayesian approach has been developed for the joint estimation of all three parameters.

The main purpose of this paper is to propose a mixed moments (MXM) method of fitting the IG3 and demonstrate the benefits of its use. This is an iterative procedure utilizing in each step a standardized version of the empirical moment generating function (EMGF) and the sample mean. Transform methods have been widely used in estimation and testing problems over the last 30 years. The EMGF was first employed in estimation problems by Quandt and Ramsey (1978) and later by Epps and Pulley (1985) and Koutrouvelis and Canavos (1997).

Section 2 briefly reviews basic properties of the IG3. Section 3 describes various existing methods for fitting this distribution and introduces the MXM method. Besides the ML method, we have considered the classical method of moments and two of its modifications. The MXM method combines the method of moments with a regression method based on the EMGF. It is an adaptation of the procedure proposed by Koutrouvelis and Canavos (1999) for fitting the Pearson type III family of distributions; see, also, Koutrouvelis and Meintanis (2002).

The asymptotic distribution of the MXM estimators is given in Section 4. In Section 5 we use Monte Carlo simulation and compare the performance of the methods of fitting the IG3 with samples of various sizes. We also compare the finite sample results and the asymptotic results for the MXM and ML methods. These comparisons show that the MXM method is more efficient than the ML and the other methods over large portions of the parameter space with small and moderate size samples and, at the same time, the distribution of the MXM estimators is close to the asymptotic distribution. It is thus possible to use the asymptotic results with small samples to find approximate confidence intervals and to draw inferences for the parameters. The MXM estimate of the scale parameter and appropriately standardized values of the EMGF may also be used in an exploratory graph for the appropriateness of the IG3 model. The procedures are illustrated in Section 6 with two numerical examples.