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On the optimality of the optimal policies for the deterministic EPQ with partial backordering

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ABSTRACT

In a note published in Omega [Zhang RQ. A note on the deterministic EPQ with partial backordering. Omega 2009;37(5):1036–8], the amended decision procedure for the Pentico et al.'s EPQ with partial backordering (EPQ-PBO) is proposed, by developing another critical value of the backordering rate. However, there is a case when a decision made with this amended procedure is not optimal, which will be shown in this paper. A new decision procedure will be proposed based on the derived necessary and sufficient conditions for considering the policy of losing all sales or the policy to meet all demand as possible optimal decisions. The proposed decision procedure is adapted for one of the extensions of the EPQ-PBO.

1. Introduction

One of the widely used models for inventory control, the classic square root economic order quantity (EOQ) model, during the past years has been the basis for many other models. When the assumption of instantaneous replenishment is replaced with the assumption that the replenishment order is received at a constant finite rate over time, EOQ is extended to the economic production quantity (EPQ) model. Pentico et al. [1] relaxed one more assumption, they allowed stockouts with partial backordering in their model, and proposed EPQ with partial backordering (EPQ-PBO). Recently, a few extensions and supplements to Pentico et al.'s EPQ-PBO have been published; some of them are Pentico et al. [2], Toews et al. [3], Wee and Wang [4].

Pentico et al. [1] determine the optimal inventory policy from the three cases: to lose all sales, to meet all demand, and to meet fractional demand, in a way that they developed the critical value β^* of the backordering rate β, below which the optimal policy is either to meet all demand or to lose all sales, and above which the optimal policy is to allow stockouts with partial backordering and meet fractional demand. Using the same notation from Pentico et al. [1], the critical value β^* is

$$\beta^* = 1 - \sqrt{\frac{2C_0C'_h}{DC_1^2}}. \qquad (1)$$

Showing that in a case when $\beta > \beta^*$ the policy of meeting fractional demand with partial backordering is not always the optimal choice, Zhang [5] amended Pentico et al.'s decision procedure [1] by developing another critical value β^{**} of the backordering rate β given by

$$\beta^{**} = \frac{PC_b(2C_0C'_h - DC_1^2)}{PDC'_hC_1^2 + DC_b(2C_0C'_h - DC_1^2)}. \qquad (2)$$

Zhang [5] then proposed the amended decision procedure for EPQ-PBO, which we will refer to as *Zhang's procedure*, and it is as follows:

1. Determine β^* and β^{**} from (1) and (2), respectively.
2. If $\beta \le \max(\beta^*, \beta^{**})$, compare the cost of meeting all demand (from the basic EPQ) with the cost of losing all sales. The optimal policy is the one with a lower cost.
3. If $\beta > \max(\beta^*, \beta^{**})$, the optimal policy is to meet fractional demand with partial backordering (defined by Pentico et al. [1]).

According to Zhang's procedure, when $\beta \le \max(\beta^*, \beta^{**})$, meeting a fractional demand with partial backordering cannot be an optimal decision. However, we will show that this is not always true. We will derive necessary and sufficient conditions for considering the policy of losing all sales or the policy to meet all demand as possible optimal decisions. A new decision procedure will be proposed based on these conditions. Our new procedure will be without comparison of costs.

One of the extensions of Pentico et al.'s EPQ-PBO is Pentico et al.'s EPQ-PBO and phase dependent backordering rate [2]. Relaxing the assumption on a constant all the time backordering rate β, they considered two phases of constant backordering rate.

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They model their decision procedure onto Pentico et al.'s decision procedure [1] by adding a comparison with cost of losing all sales when the backordering rate is above the critical value. Regarding the same model only using a different methodology, Hsieh and Dye [6] derived optimal solutions and proposed the decision procedure without comparison of costs. We will modify our new decision procedure to be applicable to this extension.

The paper is organized in the following manner. In Section 2 a numerical example is given to illustrate that decisions made with Zhang's procedure are not always optimal. A new cost comparison free decision procedure for Pentico et al.'s EPQ-PBO is proposed in Section 3. In Section 4 a new decision procedure for the extension EPQ-PBO and phase dependent backordering rate is proposed.

2. Numerical example

The example that is given in Zhang [5] illustrates that following Zhang's procedure, one can make a right optimal decision. The values of the parameters in this example are: $D=110$ units/year, $P=9200$ units/year, $C_0=\$275$/setup, $C_h=\$2.00$/unit/year, $C_b=\$0.70$/unit/year, $C_1=\$2.4$/unit. In order to show failure of Zhang's procedure to make the right optimal decision, we will give another example. The values of the parameters in our example are: $D=5000$ units/year, $P=9200$ units/year, $C_0=\$275$/setup, $C_h=\$2.00$/unit/year, $C_b=\$3.2$/unit/year, $C_1=\$0.5$/unit. Then, according to Pentico et al. [1], the value of C'_h is

$$C'_h=C_h(1-D/P)=2\cdot(1-5000/9200)=0.913043.$$

The critical values β^* and β^{**} according to (1) and (2), respectively, are

$$\beta^*=1-\sqrt{\frac{2C_0C'_h}{DC_1^2}}=1-\sqrt{\frac{2\cdot 275\cdot 0.913043}{5000\cdot 0.5^2}}=0.366171,$$

$$\beta^{**}=\frac{PC_b(2C_0C'_h-DC_1^2)}{PDC'_hC_1^2+DC_b(2C_0C'_h-DC_1^2)}$$
$$=\frac{9200\cdot 3.2\cdot(2\cdot 275\cdot 0.913043-5000\cdot 0.5^2)}{9200\cdot 5000\cdot 0.913043\cdot 0.5^2+5000\cdot 3.2\cdot(2\cdot 275\cdot 0.913043-5000\cdot 0.5^2)}$$
$$=15.0258.$$

Let $\beta=0.5$ (note that $\beta>\beta^*=0.366171$). According to Zhang's procedure $\beta\le\max(\beta^*,\beta^{**})=15.0258$, so we should compare the cost of meeting all demand

$$\Gamma^*_{EPQ}=\sqrt{2C_0C'_hD}=\sqrt{2\cdot 275\cdot 0.913043\cdot 5000}=1584.57$$

with the cost of losing all sales

$$\Gamma_{LS}=C_1D=0.5\cdot 5000=2500.$$

Since, $\Gamma^*_{EPQ}=1584.57<2500=\Gamma_{LS}$, according to Zhang's procedure, the optimal policy is to allow no stockouts.

But, if we calculate the optimal time length of the inventory cycle T^*, the optimal fill rate F^* and the value of the cost function $\Gamma(T,F)$ for $(T,F)=(T^*,F^*)$, according to the formulas given in Pentico et al. [1], we will obtain the values

$$T^*=0.395136,\quad F^*=0.865103,\quad \Gamma(T^*,F^*)=1560.55.$$

Comparing this cost of meeting fractional demand with the cost of meeting all demand, we have

$$\Gamma(T^*,F^*)=1560.55<\Gamma^*_{EPQ}=1584.57.$$

Therefore, even if $\beta\le\max(\beta^*,\beta^{**})$, the cost of meeting fractional demand is lower than the cost of meeting all demand (which should be the lowest cost according to Zhang's procedure), so Zhang's procedure failed in making optimal decision. Just for illustration, in this case, Pentico et al.'s decision procedure [1] will make a right optimal decision.

3. A new decision procedure for EPQ-PBO

With an intention to correct Pentico et al.'s decision procedure [1], Zhang [5] derived a new critical value β^{**}, defined with (2), and the condition for meeting fractional demand when $\beta>\beta^*$, where β^* is the critical value defined with (1). By taking in consideration the cost of losing all sales as a possible optimal decision when $\beta>\beta^*$, Zhang at first transformed the inequality $\Gamma(T^*,F^*)\le C_1D$ into

$$\beta\ge\frac{2C_0C'_b}{DC_1^2}-\frac{C'_b}{C'_h},\tag{3}$$

and then into $\beta\ge\beta^{**}$, without providing any details for these transformations. But, during the second transformation Zhang [5] overlooked the sign of the expression $PDC'_hC_1^2+DC_b(2C_0C'_h-DC_1^2)$, as we are going to show.

Namely, if we substitute $C'_b=C_b(1-\beta D/P)$ into (3) we will have

$$\beta\ge\frac{2C_0C_b(1-\beta D/P)}{DC_1^2}-\frac{C_b(1-\beta D/P)}{C'_h},$$

and after some algebraic transformations the last inequality is equivalent to

$$\beta(PDC'_hC_1^2+DC_b(2C_0C'_h-DC_1^2))\ge PC_b(2C_0C'_h-DC_1^2).\tag{4}$$

Now, if the expression $PDC'_hC_1^2+DC_b(2C_0C'_h-DC_1^2)>0$ then (4) implies

$$\beta\ge\frac{PC_b(2C_0C'_h-DC_1^2)}{PDC'_hC_1^2+DC_b(2C_0C'_h-DC_1^2)}=\beta^{**}$$

Otherwise, if $PDC'_hC_1^2+DC_b(2C_0C'_h-DC_1^2)<0$ then (4) implies $\beta\le\beta^{**}$. Consequently, the derivation in Zhang [5] is correct only if $PDC'_hC_1^2+DC_b(2C_0C'_h-DC_1^2)>0$.

Another observation that is worth mentioning is that if the expression $2C_0C'_h-DC_1^2>0$, then the expression $PDC'_hC_1^2+DC_b(2C_0C'_h-DC_1^2)>0$ and Zhang's procedure will make right decisions. On the other hand, the sign of the expression $2C_0C'_h-DC_1^2$ is closely related to the sign of the critical value β^* as it is shown below:

$$2C_0C'_h-DC_1^2>0\Leftrightarrow\frac{2C_0C'_h}{DC_1^2}>1\Leftrightarrow\beta^*=1-\sqrt{\frac{2C_0C'_h}{DC_1^2}}<0.\tag{5}$$

According to (5) and the above discussion, when $\beta^*<0$ then Zhang's procedure will make the right decisions. So, what is left is to correct Zhang's procedure when $\beta^*\ge 0$.

As it is shown below, in the case $\beta^*\ge 0$ the cost of losing all sales $\Gamma_{LS}=C_1D$ is not lower than the cost of meeting all demand $\Gamma^*_{EPQ}=\sqrt{2C_0C'_hD}$, and in this case to lose all sales can never be an optimal decision. We have

$$\beta^*=1-\sqrt{\frac{2C_0C'_h}{DC_1^2}}=1-\frac{\sqrt{2C_0C'_hD}}{DC_1}=1-\frac{\Gamma^*_{EPQ}}{\Gamma_{LS}}\ge 0\Leftrightarrow\Gamma^*_{EPQ}\le\Gamma_{LS}.\tag{6}$$

The last equation (6) gives the necessary and sufficient condition for considering the policy of losing all sales or the policy of meeting all demand as possible optimal decisions. It states that when $\beta^*\ge 0$ decision makers should not take into consideration the policy of losing all sales as an optimal decision (consequently, there is no need of the second critical value β^{**}), and when $\beta^*<0$ the policy of meeting all demand should not to be taken into consideration as an optimal one.

We will construct our new decision procedure upon the above reasoning given with Eq. (6) and the facts about the critical values β^* and β^{**} when related to the backordering rate β from Zhang [5]