Omega 40 (2012) 672–679

Contents lists available at SciVerse ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

Using branch-and-price approach to solve the directed network design problem with relays

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ARTICLE INFO

Article history:
Received 17 May 2011
Accepted 21 November 2011
Processed by Associate Editor Romeijn
Available online 27 December 2011

Keywords:
Network design
Relay points
Branch-and-price
Column generation
Integer programming

ABSTRACT

We present node-arc and arc-path formulations, and develop a branch-and-price approach for the directed network design problem with relays (DNDR). The DNDR problem can be used to model many network design problems in transportation, service, and telecommunication system, where relay points are necessary. The DNDR problem consists of introducing a subset of arcs and locating relays on a subset of nodes such that in the resulting network, the total cost (arc cost plus relay cost) is minimized, and there exists a directed path linking the origin and destination of each commodity, in which the distances between the origin and the first relay, any two consecutive relays, and the last relay and the destination do not exceed a predefined distance limit. With the node-arc formulation, we can directly solve small DNDR instances using mixed integer programming solver. With the arc-path formulation, we design a branch-and-price approach, which is a variant of branch-and-bound with bounds provided by solving linear programs using column generation at each node of the branch-and-bound tree. We design two methods to efficiently price out columns and present computational results on a set of 290 generated instances. Results demonstrate that our proposed branch-and-price approach is a computationally efficient procedure for solving the DNDR problem.

1. Introduction

In this paper, we study the directed network design problem with relays (DNDR). We formally define the DNDR problem as follows:

Definition 1 (*DNDR Problem*). Given a directed network $G=(N,A)$ with node set N and arc set A, and a commodity set K, introduce a subset of arcs and locate relays on a subset of nodes such that in the resulting network, the total arc cost plus relay cost is minimized, and there exists one directed path from the origin to the destination of each commodity, in which the distances between the origin and the first relay, any two consecutive relays, and the last relay and the destination do not exceed the predefined distance limit λ_{max}.

Fig. 1 gives an example of the DNDR problem, which is similar to that in [1] except that we convert their graph to a digraph. In this example, we need to determine directed paths for four commodities. The distance limit of locating relays is five. A feasible solution is to select the arcs in bold and locate a relay on node 5. Henceforth, we refer to each feasible origin-destination path with relays for each commodity as a path pattern. In addition, we denote relay pattern of a path as a set of relay nodes. In Fig. 1, the relay pattern for commodity 1 is an empty set, and relay patterns of other three commodities are all {5}. The DNDR problem can be used to model many network design problems in transportation, service, and telecommunication systems [2,3]. For example, the distance between the origin and destination of commodities is generally very large in freight transport network [4,5]. It becomes impractical for both the truck drivers and the whole transportation process to cover these long hauling distances in one trip [6,7]. This requires setting relay points along the paths for the exchange of drivers, trucks and trailers. These relay points can be used for various purposes, e.g., driver rest.

The DNDR problem generalizes the network design problem with relays (NDR), which was first introduced by Cabral et al. [1] in the context of real-life telecommunication network design project. Cabral et al. [1] define this problem over an undirected graph G' comprising node set N and edge set E. Edge $\{i,j\}$ has a length $d_{i,j}$ and a fixed installment cost $q_{i,j}$ (edge cost). Let K be the set of $|K|$ commodities, $K=\{1,2,3,\ldots,|K|\}$. Each commodity $k\in K$ has an origin s_k and a destination t_k. A fixed cost g_i is required for locating a relay at node $i\in N$ (relay cost). The objective of the NDR problem is to introduce a subset of edges and to locate relays on a subset of nodes such that in the resulting network, there exists a

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doi:10.1016/j.omega.2011.11.006

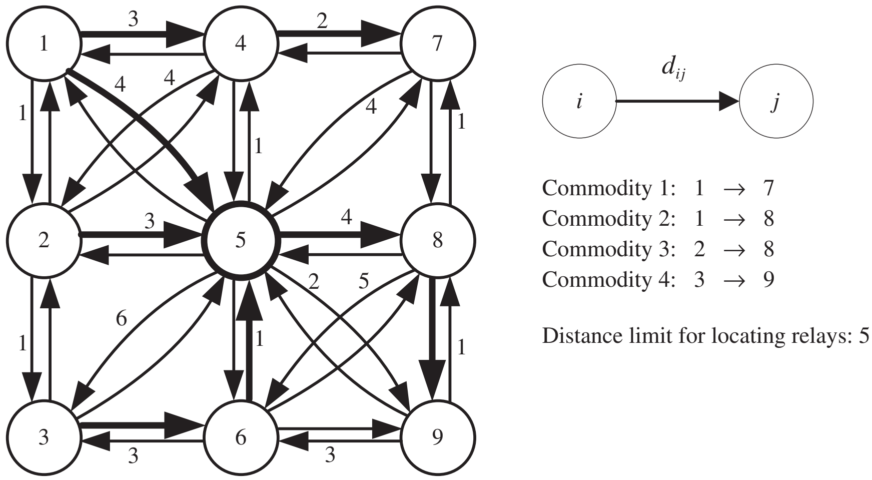

Fig. 1. Example of the DNDR problem.

bidirectional path linking the origin and destination of each commodity $k \in K$, where the distances between s_k and the first relay, any two consecutive relays, and the last relay and t_k do not exceed a predefined distance limit λ_{max}, and the total cost (edge cost plus relay cost) is minimized.

The DNDR problem covers the NDR problem as a special case. In the literature, some researchers have studied heuristic approaches for NDR problems. Cabral et al. [1] present an arc-path formulation, with an exponential number of columns known only implicitly. They first develop a lower bound (LB) procedure by solving the linear programming (LP) relaxation of the problem using a column-generation approach. Cabral et al. [1] show that sometimes this column generation based LB procedure could also optimally solve some small instances. We think this is because these instances could be easily solved at the root node of the branch and bound (B&B) tree. But note that this LB procedure is not an exact algorithm for solving NDR problems. To get integer solutions to general NDR instances, Cabral et al. [1] then propose four heuristics, of which the performance is evaluated on instances with up to 1002 vertices and 1930 edges within a time limit of 10 h on workstation class computers. The first heuristic (CH1) is a modification of the construction heuristic for Steiner tree problems, which constructs the network by considering one commodity at a time. The second and third heuristics are called increasing order heuristic (IOH) and decreasing order heuristic (DOH), respectively. The IOH constructs the network one commodity at a time in an increasing order of the path cost, but the DOH constructs the network one commodity at a time in a decreasing order of the path cost. The fourth heuristic (CH2) improves the first heuristic by using impact of adding arcs and/or relay nodes, but also significantly increase computational times. For detailed implementation of these four heuristics, see [1]. Kulturel-Konak and Konak [8] and Konak and Kulturel-Konak [9] design a metaheuristic based on genetic algorithm (GA) for the NDR problem. This algorithm was tested on 20 instances with up to 160 nodes and 3624 edges. The computational results show that the proposed metaheuristic in [8] outperforms heuristics CH1 and CH2 in [1].

The DNDR problem, NDR problem and other network design problems share similar characteristics, e.g., the Steiner tree problem, the hop-constrained network design problem [10,11], the minimum cost path problem with relays [12], and the weight constrained shortest path problem [13]. In the hop-constrained network design problem, there is a limit on the number of arcs between the origin and destination of each commodity. Voss [10] studies the Steiner tree problem with hop-constraints and proposes a tabu search based heuristic. Gouveia and Requejo [11] propose a Lagrangian relaxation approach for the hop-constrained minimum spanning tree problem. The minimum cost path problem with relays is a special case of the NDR problem, where only one commodity is considered. As an extension of the NDR problem, Konak et al. [14] study the two-edge disjoint survivable network design problem with relays, where two-edge connectivity is considered, and propose a GA based metaheuristic.

To the best of our knowledge, there is no exact algorithm in the literature for optimally solving both DNDR and NDR problems, although some heuristics have been proposed for the NDR problem. We intend to fill this void. The purpose of this paper is to develop exact branch-and-price (B&P) approaches for the DNDR problem. With mild modifications, our proposed formulations and algorithms can also be applicable to the NDR problem. We first present a new node-arc formulation, with which we can directly use mixed integer programming (MIP) solvers (e.g., CPLEX and LINGO) to efficiently solve small DNDR instances without expensive programming. Column-generation models have been extensively used in solving network flow problems formulated as linear programs with an exponential set of columns known only implicitly, e.g., multicommodity network flow problems [15]. Starting with an existing arc-path formulation, we then present a new arc-path (column-generation) formulation for the DNDR problem. With this arc-path formulation, we design a B&P approach to optimally solve the DNDR problem. In this method, sets of columns are left out of the LP because there are too many columns to handle efficiently, and most of them will have their associated variables equal to zero in an optimal solution [15]. In this column-generation formulation, we relax binary variables associated with feasible path patterns to be continuous and prove that the resulting formulation still has integer solutions. Such a relaxation allows us to use a standard branching strategy in the proposed B&P algorithms. At each node of the B&B tree, the pricing problem is solved to identify potential columns to enter the basis. If no such column is found, the LP is optimally solved. The pricing problem is a minimum cost path problem with relays (MCPR). We consider two methods to solve the pricing problem: the first one formulates the pricing problem as an MIP problem and solves it by MIP solvers, and the second one is a modified implementation of an existing labeling method for the MCPR problem. We implement and test the proposed B&P algorithm on a wide range of generated instances. Computational results demonstrate that our B&P approach is an efficient method for solving the DNDR problem.