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Discrete Applied Mathematics 148 (2005) 171–180
DISCRETE APPLIED MATHEMATICS
www.elsevier.com/locate/dam

A class of acyclic digraphs with interval competition graphs

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Received 9 April 2003; received in revised form 14 February 2005; accepted 15 February 2005
Available online 2 April 2005

Abstract

Let D be an acyclic digraph. The competition graph of D has the same set of vertices as D and an edge between vertices u and v if and only if there is a vertex x in D such that (u, x) and (v, x) are arcs of D. In this paper, we show that the competition graphs of doubly partial orders are interval graphs. We also show that an interval graph together with enough isolated vertices is the competition graph of a doubly partial order. Finally, we introduce a new notion called the doubly partial order competition number of an interval graph and present some open questions.

Keywords: Competition graph; Interval graph; Doubly partial order; Doubly partial order competition number

1. Introduction

Given a digraph $D = (V, A)$, the *competition graph* $G = C(D)$ of D has the same vertex set and has an edge xy if for some vertex $u \in V$, the arcs (x, u) and (y, u) are in D. The notion of the competition graph is due to Cohen [2] and has arisen from ecology. A *food web* in an ecosystem is a digraph whose vertices are the species of the system and which has an arc from vertex u to vertex v if and only if u prey on v. Given a food web F, it is said that species u and v compete if and only if they have a common prey. Competition graphs also have applications in coding, radio transmission, and modelling of complex economic

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doi:10.1016/j.dam.2005.02.005

systems. (See [16,18] for a summary of these applications and [6] for a sample paper on the modelling application.)

We say a graph G is an *interval graph* if it is the intersection graph of some family of intervals on the real line. Cohen [2–4] observed empirically that most competition graphs of acyclic digraphs representing food webs are interval graphs. Roberts [17] asked whether or not Cohen's observation was just an artifact of the construction and concluded that it was not by showing that if G is an arbitrary graph, then G together with as many isolated vertices as the edges of G is the competition graph of an acyclic digraph D. (Add a vertex i_α corresponding to each edge $\alpha = \{a, b\}$ of G, and draw arcs from a and b to i_α.) He then asked for a characterization of acyclic digraph D whose competition graphs $C(D)$ are interval graphs. Since then the problem has remained elusive and it has been the basic open problem in the study of competition graphs. There have been efforts at settling the problem and some progress has been made. Cohen [4] approached the problem from a statistical point of view, trying to build statistical models for the construction of D so that $C(D)$ is (likely to be) an interval graph. Steif [21] showed that there could be no forbidden subgraph characterization of acyclic digraphs whose competition graphs are interval. Lundgren and Maybee [13] gave some results which characterize such a digraph D. But these results essentially boiled down to calculating $C(D)$ and using one of the well-known (and efficient) characterizations of an interval graph. While this solves the problem, it is not what we want: a characterization in terms of properties of D. Since the general problem of characterizing acyclic digraphs whose competition graphs are interval seems difficult, Hefner et al. [8] attacked it by putting a constraint on both the indegrees and outdegrees of D.

The study on acyclic digraphs whose competition graphs are interval led to several new problems and applications. One of them is to characterize competition graphs of an interesting family of digraphs. There have been a number of papers about competition graphs of specific classes of digraphs. For instance, competition graphs of acyclic digraphs have been studied in [1,19], of arbitrary digraphs with or without loops in [1,12], of strongly connected digraphs in [5], of Hamiltonian digraphs in [5,7], of interval digraphs in [11], for various classes of symmetric digraphs in [14–16], of semiorders, of acyclic digraphs satisfying property $C(p)$, and of acyclic digraphs satisfying property $C^*(p)$ in [10]. By means of extending the results obtained in [10], Roberts (pers. comm.) proposed a problem of characterizing competition graphs of doubly partial orders. While we tried to answer his question, we could show that the competition graphs of doubly partial orders are interval graphs. We also show that an interval graph together with enough isolated vertices is the competition graph of a doubly partial order. The authors believe that these results make a significant contribution to the problem of characterizing acyclic digraphs whose competition graphs are interval.

The vast literature of competition graphs is summarized in the survey paper by Lundgren [12] and Kim [9].

2. Main results

After Kim and Roberts [10] characterized competition graphs of semiorders and interval orders, they introduced acyclic digraphs satisfying $C(p)$ and $C^*(p)$ as generalizations of