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Equivalences for Silent Transitions in Probabilistic Systems

(Extended Abstract)

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Abstract

We address abstraction in the setting of probabilistic reactive systems, and study its formal underpinnings for the *strictly alternating model*. In particular, we define the notion of *branching bisimilarity* and study its properties by studying two other equivalence relations, viz. coloured trace equivalence and branching bisimilarity using maximal probabilities. We show that both alternatives coincide with branching bisimilarity. The alternative characterisations have their own merits and focus on different aspects of branching bisimilarity. Together they give a better understanding of branching bisimilarity. A crucial observation, and, in fact a major motivation for this work is that the notions of branching bisimilarity in the alternating and in the non-alternating model differ, and that the latter one discriminates between systems that are intuitively branching bisimilar.

Keywords: Process theory, abstraction, branching bisimulation, probabilistic systems, coloured trace equivalence

1 Introduction

One of the hallmarks of process theory is the notion of *abstraction*. Abstractions allow one to reason about systems in which details, unimportant to the

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purposes at hand, have been hidden. It is an invaluable tool when dealing with complex systems. Over the past decades, research has made great strides in coping with abstraction in theories that focus on functional behaviours of systems. However, when it comes to theories focusing on functional behaviours *and* extra-functional behaviours, we suddenly find that many issues are still unresolved.

This paper addresses abstraction in the setting of systems that combine non-determinism and probabilism, hereafter referred to as *probabilistic systems*. The model we use in this paper is that of *graphs* that adhere to the strictly alternating regime as studied by Hansson [8], rather than the non-alternating model [11,12] as proposed by Segala *et al.* We study the notion of *branching bisimilarity* for this model. The need for this particular equivalence relation is already convincingly argued by Van Glabbeek and Weijland [6] and Groote and Vaandrager [7]. Recall that branching bisimilarity for probabilistic systems has been defined earlier for the non-alternating model by Segala and Lynch [12] and a variation on that notion was defined by Stoelinga [13]. We stress that the differences in the alternating model and the non-alternating model lead to incompatibilities of the notions of branching bisimilarity in both settings. In fact, these differences are an essential motivation for our investigation: while our branching bisimulation relation satisfies the properties one expects, the existing notions turn out to be too strict in their current phrasing (a more detailed account of this is given in our section on related work, see 5), and discriminate between systems that are intuitively branching bisimilar.

Van Glabbeek and Weijland [6] showed that a key property of branching bisimilarity is its preservation of *potentials* of a (non-probabilistic) system. Roughly speaking, these are the options the system has to branch and behave. They illustrated this property by defining a new equivalence, called *coloured trace equivalence*, which uses colours to code for the potentials. Subsequently, they showed that branching bisimilarity and this new equivalence coincide, and both are strictly finer than weak bisimilarity.

Although our setting is more complex than the non-probabilistic setting, the key concept of preservation of potentials should still hold. We show that this is indeed the case by defining a probabilistic counterpart of coloured trace equivalence, and show that it coincides with branching bisimilarity. A major advantage of coloured trace equivalence is that it can be understood without knowledge of probability theory and without appealing to schedulers.

Another property of branching bisimilarity (one that is due to the alternating model, and which can also be found for weak bisimilarity [10]), is the preservation of *maximal probabilities*. We show that branching bisimilarity can be rephrased in terms of such maximal probabilities.