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Fixed or variable demand? Does it matter when locating a facility? ☆

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ABSTRACT

In most competitive location models available in the literature, it is assumed that the demand is fixed independently of market conditions. However, demand may vary depending on prices, distances to the facilities, etc., especially when the goods are not essential. Taking variable demand into consideration increases the complexity of the problem and, therefore, the computational effort needed to solve it, but it may make the model more realistic. In this paper, a new planar competitive location *and design* problem with variable demand is presented. By using it, it is shown numerically for the first time in the literature that the assumption of fixed demand influences the location decision very much, and therefore the selection of the type of demand (fixed or variable) must be made with care when modeling location problems. Finally, two methods are presented to cope with the new model, an exact interval branch-and-bound method and an evolutionary algorithm called UEGO (*Universal Evolutionary Global Optimizer*).

1. Introduction

One of the major questions that a retail chain has to face when it considers entering or extending its presence in a market is 'where to locate' the new facility (or facilities) to be opened. If other facilities offering the same goods already exist in the area, the new facility will have to *compete* for the market. Many competitive location models are available in the literature, see for instance the survey papers [1–3] and the references therein. In order to evaluate the market share resulting from the entry of the new facility, one needs to consider the way consumers choose facilities offering similar goods/services. Many quite different proposals exist in the literature, as extensively explained in [4].

Observe that in most competitive location literature, it is assumed that the demand is fixed regardless the conditions of the market. Some remarkable exceptions are [5–8]. Although this may be appropriate for essential goods, in other cases this is mainly due to the difficulty of the problems to be solved: even with fixed demand, the corresponding location models may be hard-to-solve global optimization problems. However, sometimes demand is elastic, that is, it varies depending on several factors. For instance, as already stated in [5], consumer expenditures on products or services offered by the facilities may increase for a variety of reasons related to location of the new facility: opening new outlets may increase the overall utility of the product; the marketing expenditures resulting from the new facilities may increase the overall 'marketing presence' of the product, leading to increased consumer demand; or some consumers who did not patronize any of the facilities, perhaps because none were close enough to their location, may now be induced to do so. On the other hand, the quality of the facilities may also affect consumer expenditures, since a better service usually leads to more sales.

Furthermore, to our knowledge, in none of the previous studies, the effect of demand being influenced by facility layout has been investigated. The first aim of this paper is to study to what extent the optimal location and quality of new facilities to be located are affected by that assumption (see Sections 4 and 6.4). In particular, we consider a "spatial interaction model" recently proposed in the literature [9–11] to analyze this effect. The model is briefly described in Section 2. As will be shown, the corresponding problem with variable demand, introduced in Section 3, is much harder to solve, and in Section 5 we will investigate two methods to cope with it, which is the second aim of the paper. A sensitivity analysis of the model is carried out in

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Section 6. Finally, in Section 7 some conclusions and lines for future research are pointed out.

2. The continuous competitive location and design problem with fixed demand

In [9,10] a planar competitive location *and design* problem was introduced. We briefly describe it.

A chain wants to locate a new single facility in a given area of the plane, where there already exist m facilities offering the same goods or product. The first k of those m facilities belong to the chain ($0 \le k < m$). The demand is supposed to be *fixed* and concentrated at n demand points, whose locations p_i and buying power $\hat{w}_i$ are given, as well as the location f_j and quality of the existing facilities. The following notation will be used throughout this paper:

Indices

i	index of demand points, $i=1,\ldots,n$.
j	index of existing facilities, $j=1,\ldots,m$.

Variables

x	location of the new facility, $x=(x_1,x_2)$.
α	quality of the new facility ($\alpha > 0$).

Data

p_i	location of demand point i ($i=1,\ldots,n$).
$\hat{w}_i$	(fixed) demand (or buying power or total expenditure) at p_i.
f_j	location of existing facility j ($j=1,\ldots,m$).
d_{ij}	distance between demand point p_i and facility f_j.
a_{ij}	quality of facility f_j as perceived by demand point p_i.
$g_i(\cdot)$	a non-negative non-decreasing function.
u_{ij}	attraction that p_i feels for f_j (or utility of f_j perceived by the people at p_i), $u_{ij} = a_{ij}/g_i(d_{ij})$.
γ_i	weight for the quality of the new facility as perceived by demand point p_i.
$d_i^{\min}$	minimum distance from p_i at which the new facility can be located.
$\alpha_{\min}$	minimum level of quality.
$\alpha_{\max}$	maximum level of quality.
S	region of the plane where the new facility can be located.

Miscellaneous

$d_i(x)$	distance between demand point p_i and the new facility.
u_{i0}	attraction that p_i feels for the new facility, $u_{i0} = \gamma_i\alpha/g_i(d_i(x))$.
$M(x,\alpha)$	market share captured by the chain.
$F(M(x,\alpha))$	expected sales obtained by the chain.
$G(x,\alpha)$	operating costs of the new facility.
$\Pi(x,\alpha)$	profit obtained by the chain.

We assume that $g_i(d_{ij}) > 0 \ \forall i,j$. Following the framework of *spatial interaction models* introduced by Huff [12], we consider that the patronizing behavior of customers is probabilistic, that is, demand points split their buying power among the facilities proportionally to the attraction they feel for them. The attraction that a demand point feels for a facility depends on both the location of the facility and its quality, as perceived by the demand point.

Based on these assumptions the market share captured by the chain is

$$M(x,\alpha) = \sum_{i=1}^{n} \hat{w}_i \frac{u_{i0} + \sum_{j=1}^{k} u_{ij}}{u_{i0} + \sum_{j=1}^{m} u_{ij}}.$$

and the problem of profit maximization is described by

$$\begin{cases} \max & \Pi(x,\alpha) = F(M(x,\alpha)) - G(x,\alpha) \\ \text{s.t.} & d_i(x) \ge d_i^{\min} \ \forall i \\ & \alpha \in [\alpha_{\min},\alpha_{\max}] \\ & x \in S \subset \mathbb{R}^2, \end{cases} \quad (1)$$

where $F(\cdot)$ is a strictly increasing differentiable function which transforms the market share into expected sales, $G(x,\alpha)$ is a differentiable function which gives the operating cost of a facility located at x with quality α, and $\Pi(x,\alpha)$ is the profit obtained by the chain. The parameter $d_i^{\min} > 0$ is a given threshold, which guarantees that the new facility is not located on top of demand point p_i (although due to demand aggregation (see [13]) p_i is a point which usually represents a set of customers who occupy a given area). The parameters $\alpha_{\min}$ and $\alpha_{\max}$ are the minimum and maximum values, respectively that the quality of a facility may take in practice. By S we refer to the region of the plane where the new facility can be located.

In this paper we assume function F to be linear, $F(M(x,\alpha)) = c \cdot M(x,\alpha)$, where c is the income per unit of goods sold. Function G should increase as x approaches one of the demand points, since it is rather likely that the operational cost of the facility will be higher around those locations (due to the value of land and premises, which will make the cost of buying or renting the location higher). On the other hand, G should be a convex function in the variable α, since the more quality we expect from the facility the higher the costs will be at an increasing rate. We assume G to be separable, in the form $G(x,\alpha) = G_1(x) + G_2(\alpha)$, where $G_1(x) = \sum_{i=1}^{n} \Phi_i(d_i(x))$, with $\Phi_i(d_i(x)) = \hat{w}_i/((d_i(x))^{\phi_{i0}} + \phi_{i1})$, $\phi_{i0},\phi_{i1} > 0$ and $G_2(\alpha) = e^{(\alpha/\beta_0)+\beta_1} - e^{\beta_1}$, with $\beta_0 > 0$ and β_1 given values. Other possible expressions for F and G can be found in [9,11].

Notice that the location and the quality of the new facility are the variables of the problem, because although in most literature only the question of location is researched, these two features cannot be separated, see [14].

3. The variable demand model

In the previous model the demand $\hat{w}_i$ is assumed to be fixed at all demand points. Now, let us make the more realistic assumption that the demand at p_i is affected by the perceived utility of the facilities, given by the vector $u_i = (u_{i0},u_{i1},\ldots,u_{im})$. Making the simplifying assumption that *the utility is additive*, then $U_i = u_{i0} + \sum_{j=1}^{m} u_{ij}$ represents the total utility perceived by a customer at p_i provided by all the facilities. Hence, it is natural to assume that the actual demand at p_i is a function of U_i. Notice that this simplifying assumption still allows us to seek whether the optimal location and quality are affected by the type of demand, fixed or variable, in the sense that if they are affected under this assumption then they will be affected in the more general case in which the utility is not additive.

If we denote the maximum possible demand at p_i by $w_i^{\max}$, and the minimum possible demand at p_i by $w_i^{\min}$, then the actual demand w_i at p_i is a function of the utility vector u_i only through the total utility U_i, i.e., $w_i(U_i) = w_i^{\min} + incr_i \cdot e_i(U_i)$, where $incr_i = w_i^{\max} - w_i^{\min}$. Here, $e_i(U_i)$ is a non-negative and non-decreasing function of U_i that must not exceed 1 (notice that w_i cannot exceed $w_i^{\max}$). Function $e_i(U_i)$ can be interpreted as the share of the maximum possible increment that a customer decides to expend under a given location scenario.

There are different possible expressions for this. The following ones have been proposed in the literature:

1. Linear expenditures: it is assumed that $w_i^{\min} = 0$, so that $incr_i = w_i^{\max}$. In this model w_i is represented by $w_i(U_i) = w_i^{\max} \cdot$