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Trip packing in petrol stations replenishment

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ABSTRACT

This paper considers a generalized version of the trip packing problem that we encountered as a sub-problem of the petrol stations replenishment problem. In this version we have to assign a number of trips to a fleet composed of a limited number of non-identical tank-trucks. Each trip has a specific duration, working time of vehicles is limited and the net revenue of each trip depends on the truck used. The paper provides a mathematical formulation of the problem and proposes some construction, improvement and neighbourhood search solution heuristics. A set of benchmark problem instances is created in a way that reflects real-life situations and used to analyse the performance of the proposed heuristics. A real-life case is also used to further assess the proposed heuristics.

1. Introduction

In this paper we address a new trip packing problem which arises as a sub problem of the *petrol stations replenishment problem* (PSRP) that will be called hereafter the *generalized trip packing problem* (GTPP). To solve the PSRP (see Cornillier et al. [1]), we start by designing a set of trips to deliver the petrol products needed or ordered by the petrol stations and then pack these trips into subsets such that each subset can be performed within a working day by one of the available trucks. The GTPP is defined by a limited number of non-identical tank-trucks and a limited number of overtime hours allowed. The heterogeneous fleet implies that the revenue of a given trip depends on the truck used for the trip, and that some trips may not be feasible for a given truck due to the characteristics and capacity of the vehicle. The objective considered is to maximize the overall net revenue of operations which is the sum over all trips of net revenue minus the marginal cost of overtime hours required. The net revenue of a trip is its revenue minus the variable operating cost (fuel consumption and cost of driver regular hours) of the used truck. The fixed cost of trucks (depreciation and regular maintenance cost) is not taken into consideration as it is paid whether the truck is used or not. Some other objectives can be overseen. Mainly, three alternative objectives are considered: to maximize the total amount of products to deliver, to maximize overall net revenue per litre delivered, and to minimize the maximum overtime hours. However, in business the main and most used objective is to maximize overall net revenue.

To the best of our knowledge, the GTPP has never been addressed in the literature. A more simplified version of this problem, called hereafter the *simple trip packing problem* (STPP), has been mentioned in the literature dealing with the *multi-trip vehicle routing problem* (MTVRP). While some contributions try to solve the MTVRP as a whole (see Brandão and Mercer [2,3]), most others solve the problem in two steps: first generate the set of routes to be used and then pack these trips (see Fleischmann [4], Taillard et al. [5], Golden et al. [6], Zhao [7], Petch and Salhi [8] and Olivera and Viera [9]).

STPP usually assumes that we have an unlimited number of identical trucks (unlimited homogeneous fleet), that all working hours are regular hours and that trip cost and revenue are the same whichever trucks are used. Consequently, the problem is reduced to the standard *bin packing problem* (BPP). This explains why the contributions addressing the homogeneous fleet MTVRP solve the trip packing sub-problem as a bin packing problem. For a survey of the literature on bin packing problems see Coffman et al. [10].

Only Prins [11] considered the multi-trip vehicle routing problem (MTVRP) with limited heterogeneous fleet. However he also solves the trip packing sub-problem as a bin packing problem using a straightforward adaptation of the first fit decreasing (FFD) heuristic. The objective considered is to minimize the number of used vehicles. Trips' cost and revenue are not taken into consideration.

In contrast, in the GTPP, trips are packed into working days that have maximum regular and overtime working hours, trucks have different capacities and trip net revenues depend on the

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trucks used. This implies that solution methods designed to solve the bin packing problem (BPP) or the simple trip packing problem (STTP) cannot be used to solve the GTPP.

Finally, it is worth noting that in practice we have about 1 h to solve the petrol stations replenishment problem (PSRP). This should be done at the beginning of each day as petrol stations can issue orders at any time and ordered products should be delivered within the following 24 h. The solution procedure proposed in [1] is an iterative procedure where each iteration consists in designing a set of trips and then solving the corresponding GTPP. Thus we need to design very fast trip packing methods.

2. Literature review

As mentioned above, to the best of our knowledge, there is no publication that deals with the GTPP. Moreover, there are few publications that deal with the simple trip packing problem (STPP). Most of these publications use an adaptation of the BFD (best fit decreasing) heuristic to solve the packing problem. Most of these publications consider the case of homogeneous fleets.

Fleischmann [4] was the first to address the STPP as a sub-problem of the MTVRP. He proposed a modified version of the savings algorithm [12] to design a set of feasible trips and used a bin packing heuristic to assign trips to trucks. Taillard et al. [5] proposed a three-phase approach to solve the MTVRP. First they used the Tabu search heuristic of Rochat and Taillard [13] to generate a set of trips. Then they constructed a set of solutions as if each vehicle is used for only one trip. Finally they used a bin packing heuristic to allocate trips to vehicles. Golden et al. [6] solved the MTVRP with the objective of minimizing the longest trip. They used a Tabu search procedure to produce several solutions for the VRP and then used an adaptation of the BFD and an interchange procedure to assign trips to trucks.

Zhao et al. [7] proposed a Tabu search heuristic with an embedded bin packing procedure. Thus, within the Tabu search, each visited neighbour solution is evaluated only after assigning trips to vehicles. Petch and Salhi [8] developed methods to generate several solutions where each vehicle is used for only one trip and, for each solution, used a modified version of the BFD to assign trips to vehicles. The best obtained MTVRP solution is then retained. In all these contributions, the fleet is composed of an unlimited number of identical vehicles (unlimited homogeneous fleet).

Olivera and Viera [9] also decompose the MTVRP and solve the trip packing sub-problem as bin packing problem. Although they consider a limited number of identical vehicles (limited homogeneous fleet), their objective is to minimize the number of vehicles to use. They indicate that they failed to solve some of their test instances.

Finally, as mentioned above, only Prins [11] considered the MTVRP with heterogeneous unlimited fleet. He assumed that all trucks have the same maximum working hours and decomposed the problem into two sub-problems: trip construction and trip packing. He developed some heuristics to construct individual trips and suggested using an adaptation of the FFD (first fit decreasing) algorithm to pack trips into a minimum number of trucks. Trip revenue and cost are not taken into consideration.

Unfortunately, none of the above mentioned trip packing methods can be used to solve the GTPP as they are not designed to maximize the overall net revenue and often unable to pack trips for a heterogeneous limited fleet. Furthermore, these methods are not designed to handle the possibility of working overtime hours in addition to regular hours.

3. Problem formulation

This paper considers the GTPP arose in solving the petrol stations replenishment problem (see Taqa Allah et al. [14], Malépart et al. [15], Cornillier et al. [16] and Cornillier et al. [17]). This real-life problem is considered under the following assumptions:

1. there are a limited number of non-identical tank-trucks. Trucks are owned by the transportation company and, consequently, the fixed costs of trucks (depreciation and regular maintenance cost) are paid whether they are used or not;
2. in addition to regular working hours, each driver can work a limited number of overtime hours;
3. when drivers work overtime, they are paid a number of hours equal to the smallest integer larger than or equal to real overtime worked;
4. for each truck, some trips are feasible and others are not depending on truck characteristics and capacity;
5. trip revenue and variable cost depends on the truck to be used for the trip but trip duration is the same whichever truck used.

The following notation will be used to formulate the generalized trip packing problem (GTPP) considered in this paper:

Parameters:

T set of trips to be packed; $|T|=n$
J set of available trucks; $|J|=m$
i trip index; $i\in T$
j truck index; $j\in J$
L maximum number of regular work hours
M maximum number of overtime hours
C marginal cost per overtime hour
d_i duration of trip i (in hours)
J_i set of trucks that can perform trip i, $|J_i|=m_i$
T_j set of trips that can be carried out by truck j
V_{ij} net revenue (revenue minus variable operating cost) of trip i if assigned to truck j

Decision variables:

x_{ij} binary equals 1 if trip i is assigned to truck j
O_j overtime hours for truck j

Using this notation, the generalized trip packing problem can be formulated as follows.

$$\text{Find}: \quad O_j \text{ integer} \geq 0 \text{ and } x_{ij} \in \{0,1\}, \quad \forall i \in T \text{ and } \forall j \in J \text{ which}:$$

$$\text{Maximize}: \quad Z=\sum_{i \in T}\sum_{j \in J_i} V_{ij}x_{ij} - C\sum_{j \in J} O_j$$

$$\text{Subject to}: \quad \sum_{j \in J_i} x_{ij} = 1, \quad \forall i \in T$$

$$\sum_{i \in T_j} d_i x_{ij} - O_j \leq L, \quad \forall j \in J$$

$$O_j \leq M, \quad \forall j \in J$$

In this formulation, the objective is to maximize the overall net revenue of operations (sum over all trips of net revenue minus the marginal cost of overtime hours). Notice that O_j should take an integer value (see assumption 3). The first set of constraints assigns each trip to one and only one truck. The second set allows determining the overtime hours for each truck and the third set prevents overtime hours from exceeding the given upper limit.

Proposition. *The generalized trip packing problem (GTPP) is NP-hard.*