Available online at www.sciencedirect.com
SCIENCE DIRECT®

Electronic Notes in Theoretical Computer Science

Electronic Notes in Theoretical Computer Science 128 (2005) 69–85
www.elsevier.com/locate/entcs

Games for Counting Abstractions [1]

J.-F. Raskin†, M. Samuelides‡ and L. Van Begin†[2]

† *Computer Science Department, University of Brussels, Belgium*
‡ *Ecole Normale Supérieure de Cachan, France*

Abstract

It has been recently shown that monotonicity is not sufficient to obtain decidability results for two-player games. However, positive results can be obtained on restricted subclasses of monotonic two-player games. In this paper, we identify and study a subclass of monotonic two-player games that is useful for analysis of parametric (open) systems that can be modeled by using counting abstractions. Although the reachability game problem is undecidable in general for that subclass, we identify two interesting and decidable problems and show how to apply those results in parametric system analysis.

Keywords: Petri nets, Counting Abstraction, Games

1 Introduction

Model-checking methods were originally proposed for the automatic verification of critical systems that have natural finite-state abstractions. Nevertheless, much *recent* interest has concerned the application of model-checking methods to infinite-state systems. Several interesting classes of infinite state systems have been shown to be decidable. For example, Alur et al. [3] showed that timed automata have a decidable *reachability problem*. Finkel et al. in [6], and Abdulla et al. in [2] have shown that infinite, but *monotonic*, transition systems (also called well-structured transition systems) have a decidable *coverability problem*. For instance, Petri nets and broadcast protocols define

[1] Supported by the FRFC project "Centre Fédéré en Vérification" funded by the Belgian National Science Fundation (FNRS) under grant nr 2.4530.02.
[2] Supported by a "First Europe" grant EPH3310300R0012 of the Walloon Region.

doi:10.1016/j.entcs.2005.04.005

monotonic transition systems.

Timed automata, Petri nets, and broadcast protocols are usually used to model *reactive systems* embedded in a critical environment. But those formalisms define transition systems that are semantic models for *closed systems*. In closed systems, we do not distinguish between the reactive system and its environment. So the properties that we can verify on transition systems are properties in which we cannot distinguish between the role of the reactive system and the role of the environment. If we want to distinguish the role of the reactive system and the environment in which it is embedded, we can use games played on state spaces.

Usual transition systems can be considered as one-player games on which only closed-system verification problems can be formulated. The *control* and *modular verification* problems of systems can be studied as two-player games played on state spaces, where *one player*, say player 1, represents the reactive system and the *other player*, say player 2, represents the environment. If the state space on which the game is played is infinite then we have to solve infinite-state games. Infinite-state games have not yet been studied as intensively as traditional verification problems on infinite-state transition systems. Nevertheless, recently there have been several interesting works in that direction. Here are some examples. In [11], Maler et al. study how to solve games defined by timed automata. In [15], Walukiewicz studies how to solve infinite games defined by pushdown automata. In [4], Henzinger et al. study symbolic algorithms to solve general infinite-state games.

In this paper, we study two-player games played on infinite but *monotonic game structures*. In particular, we study games that are useful to study *parametric systems*. Parametric systems are systems where the number of instances of component types is not fixed a priori. In general, we are interested to verify such systems for any number of instances. The notion of *counting abstraction* [8] is a powerful tool to reason on parametric systems and consists in only retaining, for each component type, the number of instances that are in each possible (local) configuration. Hence, if the number of component types and the number of (local) configurations for each component is finite, then states of parametric systems can be abstracted by integer vectors. In that context, parametric systems are modeled as vector addition systems with states (VASS for short). Here, we want to consider parametric systems as open systems over which game properties can be formulated and verified. For that, we identify an interesting *subclass* of monotonic game structures for which the *coverability game* and the *deadlock-avoidance game* problems are decidable. A restricted form of two-player VASS systems define game structures that fall into that class. We illustrate the interest of our decidable game structures