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A modified DEA model to estimate the importance of objectives with an application to agricultural economics

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ABSTRACT

This paper demonstrates a connection between data envelopment analysis (DEA) and a non-interactive elicitation method to estimate the weights of objectives for decision-makers in a multiple attribute approach. This connection gives rise to a modified DEA model that allows us to estimate not only efficiency measures but also preference weights by radially projecting each unit onto a linear combination of the elements of the payoff matrix (which is obtained by standard multicriteria methods). For users of multiple attribute decision analysis the basic contribution of this paper is a new interpretation in terms of efficiency of the non-interactive methodology employed to estimate weights in a multicriteria approach. We also propose a modified procedure to calculate an efficient payoff matrix and a procedure to estimate weights through a radial projection rather than a distance minimization. For DEA users, we provide a modified DEA procedure to calculate preference weights and efficiency measures that does not depend on any observations in the dataset. This methodology has been applied to an agricultural case study in Spain.

1. Introduction and objectives

Several authors have pointed out some close connections between data envelopment analysis (DEA) and multicriteria decision making (MCDM): see Belton and Vickers [1], Steward [2,3], Zhu [4], Joro et al. [5], Chen [6], Bouyssou [7], Andre [8]. Some of these authors have underlined the equivalence between the notion of "efficiency" in DEA and MCDM (e.g. [6,7, p. 974]) although the two approaches are different regarding how efficiency is measured in practise. In DEA, the so-called "efficient frontier" is built as the envelope of all the decision making units (DMUs hereafter) included in the sample. Efficiency is, therefore, measured in relative terms by comparing each unit with the others in the same sample. On the contrary, in MCDM, efficiency is measured in absolute terms. That is, in a MCDM problem, the decision-maker (DM) faces a number of constraints which determines the feasible set. Therefore, by exploring the feasible set it is possible to determine which solutions are efficient or not (and hence, which DMs adopting those solutions behave efficiently), without any comparison across DMs. Translating multicriteria objectives into DEA terminology, a "*max*" objective can be understood as an output whereas a "*min*" objective can be interpreted as an input or a bad output [2,7,9].

We report a further connection by stressing the parallelism between DEA and the multicriteria non-interactive method proposed by Sumpsi et al. [10] to estimate the weights of different objectives in the preferences of DMs. We claim that, although these methodologies have been developed independently of each other, there is a strong parallelism between them. The *first contribution* of this paper is to underline this connection between DEA and this MCDM methodology, as well as providing a new interpretation for the procedure of Sumpsi et al. in terms of efficiency.

MCDM and DEA also have in common that both of them deal with individuals, activities or organizations that are concerned with multiple objectives or inputs and outputs. In such a framework, it would appear to be relevant to measure or evaluate the relative importance of each objective, input or output according to the preferences of DMs. As we will discuss in Section 3, the methodology of Sumpsi et al. is aimed at measuring this importance by projecting the observed values of objectives onto a linear combination of the elements of the payoff matrix (where such a matrix is obtained by optimizing each objective separately). We claim that, provided that all the elements of the payoff matrix are efficient, the procedure introduced by Sumpsi et al. has a strong resemblance to DEA, where each unit is projected onto a combination of efficient units. In order to guarantee that the elements of the payoff matrix are efficient we propose to construct the payoff matrix by solving an auxiliary lexicographic problem.

On the other hand, although the aim of DEA is to estimate not preferences but efficiency scores, it requires the construction of a weighted combination of inputs and outputs. As the weights

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(known as *virtual multipliers*) used to compute such combinations are endogenously determined to provide the best possible score for each unit, they could be understood as having some connection with the preferences of DMs. For example, Cooper et al. [11] suggest bounding DEA weights according to the importance given by some experts to each of the criteria (inputs) using an analytic hierarchy process (AHP) analysis. However, the weights obtained from a standard DEA analysis are not a suitable measure of the preferences of a given DM, since DEA parameters are crucially influenced by the structure of the production process under analysis, which is often related to technological issues and not to the preferences of decision-makers. Moreover, the representation of the efficient frontier in DEA is critically influenced by the amounts of inputs and outputs of other observations in the dataset whereas, in principle, the preferences of an individual should not be influenced by the decisions of other individuals.

The *second contribution* of this paper is to establish a particular way to apply DEA in order to obtain estimates of preference parameters, by taking advantage of the parallelism between DEA and the Sumpsi et al. methodology. For this purpose, we propose to project radially each decision unit onto a linear combination of the (efficient) elements of the payoff matrix. The main idea is to use DEA including the elements of the payoff matrix as the only units in the reference set and to interpret the parameters associated to each reference unit (denoted as λ) as the weights given by the DM to each criterion or throughput. The underlying rationale of this procedure is to control for the technological constraints (those related to the production structure) and isolate the effects specifically associated with preferences. By evaluating the distance to each element of the payoff matrix it can be inferred which criteria are revealed as more or less important for the DM. Using this approach, we arrive at both an estimation of the preference weights for each DM and an approximate measure of efficiency in a single model. This efficiency measure has the property of being independent of the rest of the observations in the dataset. A key advantage of using this modified DEA model rather than the methodology proposed by Sumpsi et al. is that, when using a DEA-like approach, the projected points on the efficient frontier keep the same proportion of inputs and outputs than the real observations to which they are associated. In this sense, the projected points can be seen as being *more similar* in their preferences to the original observations they come from.

Our methodological proposal has some resemblance with the idea introduced by Golany and Roll [12], Cook et al. [13] and Cook and Zhu [14] which consists in including "standards" into the sample of DMUs to get so-called benchmarking DEA models. The common feature is that we propose to use as reference units the elements of the payoff matrix, which might be seen as a particular kind of standards. Nevertheless, there are also important differences. First, from the technical point of view, our reference set consists only of the elements of the payoff matrix, whereas the aforementioned references include the standards together with the sample of observed DMUs. More importantly, the goal of both approaches is different. In Golany and Roll [12] and Cook and Zhu [14], the motivation to include the standards is to improve the measurement of efficiency whereas, in our case, the aim is to get preference weights estimates and (approximate) efficiency measures are obtained only as a by-product. Bougnol et al. [15] affirmed that it is a common and unconscious practise in real world to include these standards to measure performance without formally applying a DEA model. Ulucan and Atici [16], noted that in some cases the benchmarks for some units were not realistic and they suggested a different approach to improve efficiency measures based on clustering techniques.

The paper has the following structure: Section 2 reviews the basic elements of the DEA approach. Section 3 presents the Sumpsi et al. methodology and proposes a modification to guarantee that all the elements of the payoff matrix are efficient. Section 4 stresses the connections between both methodologies and, using these connections, it presents an alternative way of using DEA to measure efficiency and estimate the weights of inputs and outputs. Section 5 presents an empirical application of the suggested method to agricultural economics, using real data from an irrigated area in Spain. In the light of the results, this paper not only demonstrates a connection between two different methodologies, but also proposes a model that will provide the results of the two methodologies at the same time. On the one hand, we obtain efficiency measures that are very close to the real values and to conventional DEA measures. Moreover, these efficiency measures have the advantage of being determined by the structure of the feasible set alone, and not by the other elements in the dataset. As a result, the efficiency score for each unit is robust with respect to any change in the sample. On the other hand, we also obtain preference weights that are very similar to those obtained when using the methodology of Sumpsi et al. Since, following a DEA logic, these weights are obtained from a radial projection, they have the peculiarity that the proportions among the relevant objectives in the observed data and the projection are the same. In order to test the practical usefulness of these estimates we show, in a validation exercise, that they provide a good approximation to observed behaviour. Section 6 summarizes the main contributions of the paper.

2. DEA model

In a standard DEA model there are n DMUs, using s different inputs to produce t different outputs. The envelopment DEA model proposed by Banker et al. [17] can be formulated as follows:

$$\begin{aligned}
&Max \quad \theta\\
&s.t.: \quad \lambda^T Y \geq \theta Y_0\\
&\qquad\quad \lambda^T X \leq X_0\\
&\qquad\quad \vec{1}\lambda = 1\\
&\qquad\quad \lambda \geq 0
\end{aligned} \tag{$BCC_E - O$}$$

where $X(Y)$ is the matrix representing all the inputs (outputs) of all the DMUs, T denotes transposing, $\vec{1} \equiv (1, \ldots, 1)$ and the λ_j parameters $(j = 1, \ldots, n)$ are the weights associated with each observed DMU in order to construct a convex combination of all of them (or just a subset if some λ_j's are equal to zero). The values of these parameters are DMU-specific.

DEA seeks to identify efficient units and combine them to construct an *efficient frontier*. A unit is said to be *radially* efficient if the optimal value of θ is equal to one. In order to guarantee that a unit is fully efficient, a second phase analysis must be carried out. In this second optimization stage the sum of the positive and negative slacks, defined as $s^+ = \lambda^T Y - \theta Y_0$ and $s^- = X_0 - \lambda^T X$ respectively, is maximized. In this case, a unit is said to be *fully* efficient if the optimal value of θ is equal to one and all the slacks are equal to zero. The technical efficiency rate (TE) is given by $TE = 1/\theta$, which is upper bounded by one and lower bounded by zero.

The *peer units* associated with the unit under analysis are those with a strictly positive value of λ. The combination (weighted by the λs) of these peer units defines a virtual unit on the frontier that is the efficient projection of the unit under analysis.