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Fire scheduling for planned artillery attack operations under time-dependent destruction probabilities

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ABSTRACT

In this paper, we consider the fire scheduling problem (FSP) for field artillery, which is the problem of scheduling operations of firing at given targets with a given set of weapons. We consider a situation in which the number of available weapons is smaller than the number of targets, the targets are assigned to the weapons already, and targets may move and hence the probability that a target is destroyed by a firing attack decreases as time passes. We present a branch and bound algorithm for the FSP with the objective of minimizing total threat of the targets, which is expressed as a function of the destruction probabilities of the targets. Results of computational tests show that the suggested algorithm solves problems of a medium size in a reasonable amount of computation time.

1. Introduction

In this paper, we consider a scheduling problem arising in the military, especially in artillery operations. There are two types of artillery operations, planned artillery operations and responsive artillery operations. In a planned artillery attack operation, a set of targets, or enemy units, is to be fired at and destroyed by weapons such as field artillery units according to a predetermined operation plan. On the other hand, in responsive artillery attack operation, enemy units are fired at (upon real-time requests of friendly units) in real-time combat situations. We consider the planned artillery attack operation. In this paper, a *weapon* represents an artillery battery, which is the basic unit to execute artillery attack operations and is typically composed of six howitzers, and a *target* represents enemy unit(s), i.e., armor unit(s), infantry unit(s), transportation unit(s), etc. In general, targets are identified by reconnaissance assets, such as human intelligence units, unmanned aerial vehicles and artificial satellites, and it is assumed in this research that locations of the targets are known.

The planned artillery attack operation consists of two steps of a sequential procedure: assignment of weapons to the targets; and scheduling firing operations against the targets that are assigned to each weapon. Note that when there are more targets than available weapons, one weapon may have to be assigned to more than one target. The decision problems of the first step and the second step are called the weapon target assignment problem (WTAP) as defined in Ahuja et al. [1] and the fire scheduling problem (FSP), respectively.

This paper focuses on the FSP, the problem of scheduling operations of firing at given targets with a given set of weapons for the objective of minimizing the threats from the enemies that survive the firing attack. In this problem, n targets are to be fired at and destroyed by m weapons. We assume that firing operations are already assigned to the weapons. For a firing operation, a prespecified set of weapons should start firing at a target simultaneously. We consider a situation in which the number of weapons is smaller than the number of targets ($m < n$), and hence the sequence of targets to be fired at should be determined. In such a situation, while a set of targets is under firing attack, other targets are usually alerted by a situation awareness system and these targets may move to avoid an anticipated incoming attack. Note that many types of targets, such as tank units, self-propelled artillery and combat support units, have mobility to hide themselves from attacks. This will decrease the probability that these targets are destroyed by the firing attack. In this paper, we assume the destruction probability decreases linearly as time passes. In the following, we briefly review research results on the WTAP, FSP and other related problems.

Reviews on earlier research articles on the WTAP are given in Matlin [2] and Eckler and Burr [3]. Manne [4] first considers the WTAP and solves the problem after transforming the problem into a transportation problem, and Lloyd and Witsenhausen [5] show that the WTAP is NP-complete. Later, Hosein and Athans [6] consider a multi-stage or dynamic version of the WTAP, and present a method in which weapons are assigned to the targets in a period based on the assignment made in the previous period. In addition, Christ et al. [7] and Erdem and Ozdemirel [8] present a

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solution method based on neural networks and a genetic algorithm, respectively, while Lee and Lee [9] give a hybrid search algorithm composed of an ant colony optimization algorithm and a genetic algorithm. On the other hand, Ahuja et al. [1] propose exact and heuristic approaches to the WTAP. Unlike most of the research, in which weapons are to be allocated without consideration of the behavior of opponents or enemy troops, Malcolm [10] and Karasakal [11] present models in which the opponent's behavior is taken into account by assuming friendly forces are equipped with a situation awareness system. The model for the WTAP is also used in the advertisement industry for media and budget allocation [12], and in the medicine area for cancer modeling [13].

Unlike the WTAP, research on the FSP is very rare. Introducing the FSP first, Kwon et al. [14] consider the FSP with the objective of minimizing makespan under the assumption that the targets are fixed (do not move) and the destruction probability is constant over time. Note that they focus on achieving a surprise attack effect through the quickest completion of the firing operation. However, their algorithm may not be applicable to cases in which the targets have mobility and are equipped with an alert system.

In most machine scheduling problems considered in the literature, it is generally assumed that a job can be processed on one and only one machine [15–17]. However, it is not the case with the problem considered in this study. The FSP is similar to the simultaneous resource scheduling problem [18], or the multiprocessor task scheduling problem (MPTSP) [19], in which each task requires one or more processors simultaneously. However, in the FSP, different weapons may require different firing durations, i.e., processing times for the firing operation, even for the same target, while an operation requires the same processing time on the machines in these scheduling problems. In other words, these scheduling problems are special cases of the FSP. Since it is proven that the MPTSP is NP-hard [19], the FSP is also NP-hard. As surveyed by Drozdowski [20], Lee et al. [21] and Allahverdi et al. [22], various solution methods have been developed for these scheduling problems [23–28].

In this research, we consider the FSP with the objective of minimizing total threat of targets or opponents. The threat of a target is expressed as a function of the destruction probability, which depends on the weapon-target pair. It is assumed that the destruction probability decreases linearly as time passes. We develop several dominance properties and lower bounds for the FSP and present a branch and bound (B&B) algorithm using them. Due to the complexity of the problem and the requirement of prompt decisions in practice, we also suggest a heuristic algorithm that can give reasonably good solutions in a short time. The heuristic algorithm is also used in the B&B algorithm for an initial upper bound. In the next section, we describe the FSP considered in this study in more detail.

2. Problem description

The fire scheduling problem (FSP) considered in this study is the problem of scheduling a set of firing operations with the objective of minimizing total threat from the targets, or opponents, that survive the firing attack. We consider two types of targets in this research, single- and multiple-weapon targets. *Single-weapon targets*, also called point targets, have small target areas and hence a single weapon is assigned to a point target. On the other hand, *multiple-weapon targets*, or area targets, cannot be handled by a single weapon, and hence multiple weapons are assigned to an area target. That is, each area target is partitioned into sub-areas and a weapon is assigned to each sub-area. In general, these partitioned targets should be fired at simultaneously to achieve tactical surprise, and this type of firing is called the *time on target firing*.

As stated earlier, a weapon represents an artillery battery which is equipped with howitzers. In general, the enemy under a firing attack cannot see the weapon, and hence immediate counter firing from the enemy against the weapon is almost impossible. In addition, the maximum ranges of the howitzers are between 11 and 30 km while the aggressive targets such as tanks have a shorter range of less than 10 km. Therefore, we assume that the threat from a direct counter firing attack from the enemy is negligibly small during the planning horizon, which is less than 1 h in our problem as in real planned artillery operations.

In this study, a firing operation is specified by a pair of a weapon and a target associated with the operation and the duration for the operation. A set of firing operations against one target is called a *job*. A job for a single-weapon target is composed of only one firing operation, while a job for a multiple-weapon target is composed of two or more firing operations. Note that a job corresponds to a target and that an operation corresponds to a pair of a target and a weapon. We use the *period* as the unit of time lengths, that is, the firing duration for a firing operation is given as an integer multiple of the length of a period, as in real situations. By scheduling the firing operations in terms of the periods, one can synchronize firing operations of multiple weapons more easily. In typical situations, a period represents 1 min.

Before a firing attack from a weapon is started against a target, a setup operation needs to be performed on the weapon for adjustment of azimuth, firing range, etc. In general, setup times (times required for setup operations) on different weapons may be different, but the setup times on the same weapon for firing operations against different targets are the same. Since the sequence of the firing operations are predetermined (through the solution of the FSP considered in this study) in planned artillery attack operations, the setup operations for the first firing operation on each weapon is assumed to have been done by the commencement of the attack, i.e., by time 0 in the planning horizon. Therefore, in this study, we assume the setup time for the first firing operation of each weapon is 0.

The following are additional assumptions made in this study.

(1) The locations and types of the targets are known, and threats from the targets may be different for different targets.
(2) The targets are already allocated to the weapons.
(3) The duration of the firing operation for each weapon-target pair as well as the setup time on each weapon is given, and they are integer multiples of the length of a period.
(4) Each weapon can fire at no more than one target in a single period.
(5) Preemption of a firing operation is not allowed.
(6) The destruction probability associated with a firing attack decreases linearly as time passes after the first firing operation is started (since the targets, which are movable and equipped with a situation awareness system, hide themselves from anticipated attack).
(7) Counter firing attacks from the enemy is not expected during the planning horizon.
(8) Firing operations against targets assigned to more than one weapon, such as multiple-weapon targets, should be started simultaneously from the weapons (to achieve surprise effect). The end times of the operations from different weapons may be different.

In the following, we present a mathematical formulation of the FSP. In the formulation, the following notation is used.