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A branching method for the fixed charge transportation problem

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ABSTRACT

This paper presents a branching method for the solution of the fixed charge transportation problem. Starting with a linear formulation of the problem, we develop the method which converges to the optimal solution. The method is based on the computation of a lower bound and an upper bound embedded within a branching process. We present a detailed numerical example to illustrate the proposed method.

1. Introduction

The fixed charge transportation problem (FCTP) is a special type of integer programming problem in which a fixed cost, sometimes called a setup cost, is incurred if a distribution variable assumes a nonzero value. The literature provides only few exact methods for solving the FCTP. The method of ranking extreme points requires analyzing a large domain of load distributions. The size of the domain depends on the initial distribution. Murty [19] based the initial distribution on the optimal distribution to the variable portion of the problem, which he admitted to be a weak start for problems with large fixed charge values. Sadagopan and Ravindran [22] improved the procedure with a dual approach by solving the fixed charge portion of the problem and switching to it if it gives a better result than Murty's method. As they demonstrated, their technique provides a considerable improvement over Murty but still requires analyzing a sizable domain of distributions.

One approach to solving FCTP involves a mixed integer programming formulation. Gray [12] attempted to find an exact solution to this problem by decomposing it into a master integer program and a series of transportation sub-programs. In contrast, Palekar et al. [21], and Steinberg [24] provided exact algorithms based on the branch-and-bound method. The exact branch-and-bound method is applicable to small problems only, since the effort to solve an FCTP grows exponentially with the size of the problem. Finding a solution to a 4×4 problem may require analyzing several hundred distributions; see for example [27]. Ortega and Wolsey [20] provided a branch-and-cut method for uncapacitated fixed charge network flow problems.

Many authors [8,9,11,13,25,27] have turned to efficient heuristic algorithms for solving FCTP because the methods are constrained by limits on computer time. Other well-known heuristic approaches are presented by Diaby [10], and Kuhn and Baumol [17]. Adlakha and Kowalski [2] present a simple heuristic based on Hungarian and Vogel approximation methods which is only useful for small problems. Adlakha et al. [4] presented a series of heuristics by continually improving accuracy while computational time increased. Some authors [3,23] have presented algorithms for the solution of fixed charge problems where the fixed charge is associated with the supply points instead of the routes. Some papers [5,15] presented other approaches to the FCTP while treatment of special structure transportation problems can be found in [1,7,16].

In this paper we develop an analytical branching method to solve the FCTP. The method starts with a linear formulation of the problem which converges to an optimal solution by sequentially separating the fixed costs and finding a direction to improve the value of the linear formulation. The iterative procedure continually tightens the lower and upper bounds as it progresses. The optimum solution is achieved when the two bounds are matched. This method is an iterational/convergence method unlike classical branch and bound which go through all distributions with some improvements like excluding some branches, limiting solution to some areas [18,20,26–28]. We present a numerical example to illustrate the steps of the proposed method.

2. Fixed charge transportation problem

The fixed charge transportation problem can be stated as a distribution problem in which there are m suppliers and n customers. Each supplier i can ship to any customer j at a shipping

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cost per unit c_{ij} plus a fixed cost f_{ij}, assumed for opening this route. Each supplier i has a_i units of supply, and each customer j has a demand for b_j units. The objective is to determine which routes are to be opened and the size of the shipment on those routes, so that the total cost of meeting demand, given the supply constraints, is minimized.

Standard FCTP formulation:

$$\mathbf{P}: \text{ Minimize} Z = \sum_{i=1}^{m}\sum_{j=1}^{n}(c_{ij}x_{ij}+f_{ij}y_{ij}) \tag{1}$$

$$\text{Subject to} \sum_{j=1}^{n} x_{ij} = a_i \quad \text{for } i=1,2,\ldots,m, \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = b_j \quad \text{for } j=1,2,\ldots,n, \tag{3}$$

$$x_{ij} \geq 0 \quad \text{for all } (i,j),$$

$$y_{ij} = 0 \quad \text{if } x_{ij} = 0,$$

$$y_{ij} = 0 \quad \text{if } x_{ij} > 0.$$

Without loss of generality, we assume that

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \quad \text{and} \quad a_i, b_j, c_{ij}, f_{ij} \geq 0.$$

Hirsch and Dantzig [14] established that the feasible region of the FCTP is a bounded convex set with a concave objective function. An optimal solution occurs at an extreme point of the constraint set, and for a non-degenerate problem with all positive fixed costs, every extreme point of the feasible region is a local minimum.

2.1. A linear formulation of the FCTP

Balinski [6] observed that there exists an optimal solution to the relaxed version of the FCTP (formed by relaxing the integer restriction on y_{ij}), with the property that

$$y_{ij} = x_{ij}/m_{ij}, \tag{4}$$

where

$$m_{ij} = \min(a_i, b_j). \tag{5}$$

So, the relaxed transportation problem (RTP) of an FCTP would be simply a classical TP with unit transportation costs as $C_{ij}=c_{ij}+f_{ij}/m_{ij}$. We refer to this problem as P'. The optimal solution $\{x'_{ij}\}$ to the RTP problem P' can be easily modified into a feasible solution of $\{x'_{ij}, y'_{ij}\}$ of P as follows:

$$y'_{ij} = 0 \text{if } x'_{ij} = 0,$$

and

$$y'_{ij} = 1 \text{if } x'_{ij} > 0.$$

Balinski shows that the optimal value of RTP, $Z(P')$, is close to the optimal solution of the corresponding FCTP and provides a lower bound on the optimal value $Z^*(P)$ of FCTP. Solution (x'_{ij}, y'_{ij}), being a feasible solution, provides an upper bound on $Z^*(P)$. Thus

$$Z(p') = \sum\sum C_{ij}x'_{ij} \leq Z^*(p) \leq \sum\sum(c_{ij}x'_{ij} + f_{ij}y'_{ij}). \tag{6}$$

Remark 1. Since the objective function of an FCTP is discrete, the lower bound in inequality (6) from the optimal solution of the RTP can be rounded up to the nearest interval.

3. The FCTP branching method

In this section we propose a branching method for solving the FCTP. The method involves first formulating and solving the Balinski RTP formulation of the problem. Next, a particular cell is chosen according to certain selection criteria. The method branches out progressively with the options of loading or excluding the chosen cell in search of an optimal solution for the FCTP.

Consider an FCTP with cost coefficients c_{ij}, fixed costs f_{ij}, and m_{ij}. The steps of the FCTP branching method are as follows:

The iterative steps

Step 1. Formulate Balinski RTP matrix with $C_{ij}=c_{ij}+f_{ij}/m_{ij}$.

Step 2. Solve as regular TP and identify the load as x'_{ij}.

Step 3. Check for the terminating conditions for a specific branch as follows:

a. If $\sum\sum C_{ij}x'_{ij} = \sum\sum(C_{ij}x'_{ij} + f_{ij}y'_{ij})$ OR
b. If $\sum\sum C_{ij}x'_{ij} \geq$ lowest recorded Z thus far, STOP and terminate this branch. Otherwise continue.

The procedure terminates when all possible branches are terminated.

Step 4. Pick out partially loaded cells, i.e., cells with $0 < x'_{ij} < m_{ij}$.

Step 5. For each cell identified in Step 4, calculate $\Delta = f_{ij} - (f_{ij}/m_{ij})x'_{ij} = f_{ij}(1 - x'_{ij}/m_{ij})$.

The Δ value, $f_{ij}(1 - x'_{ij}/m_{ij})$, represents the fixed cost differential between the RTP and the FCTP for load x'_{ij} at cell (i, j).

Step 6. Pick out the cell (s,t) with the highest Δ among those identified in Step 5.

Break ties by choosing the Δ with the largest f_{ij}. If there is more than one Δ then pick one at random.

Branch $Y(s,t)$: *load cell* (s,t)

Step 7. Note down $Z_{Ycost}=f_{st}$, where Z_{Ycost} represents the extracted fixed charges along branch $Y(s,t)$. Set $f_{st}=0$ in the FCTP.

Step 8. Repeat Steps 1–6 with the adjusted FCTP.

Branch $N(s,t)$: *exclude cell* (s,t)

Step 9. Set $c_{st}=M$ (a very large number) in the FCTP.

Step 10. Repeat Steps 1–6 with the adjusted FCTP.

Note that after Step 6 the method branches out in two directions. In Branch Y, the selected cell is loaded. The associated fixed cost is extracted as Z_{Ycost} to be combined in the objective function later and f_{st} is set to 0 in the current FCTP. In Branch N the selected cell is excluded by setting the corresponding cost to a large number.

The branches are labeled as Y, N in the first iteration; YY, YN, NY, NN in the second iteration, etc., by dividing and extending each branch into two branches and affixing Y or N with the previous branches, where Y represents the branch where the selected cell is loaded and N represents the branch where the selected cell is excluded.

The branching method exploits the relationship presented in inequality (6), which provides a lower bound and an upper bound for the FCTP under consideration. The method sequentially searches for the fixed costs f_{ij} of the given FCTP that can be extracted in an effort to induce linear properties into the problem and to search for an optimal solution by sequentially increasing the lower bound obtained by the Balinski formulation. Since the branching goes through all points in the set, the proposed method guarantees that at least one of the points is optimum, identified by the lowest upper bound.

Theorem 1. *The branching method provides a non-decreasing lower bound of an FCTP at each iterative step along a branch.*