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Relations between Gold-style learning and query learning

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Abstract

Different formal learning models address different aspects of human learning. Below we compare *Gold-style learning*—modelling learning as a *limiting process* in which the learner may change its mind arbitrarily often before converging to a correct hypothesis—to *learning via queries*—modelling learning as a *one-shot process* in which the learner is required to identify the target concept with just one hypothesis. In the Gold-style model considered below, the information presented to the learner consists of positive examples for the target concept, whereas in query learning, the learner may pose a certain kind of queries about the target concept, which will be answered correctly by an oracle (called teacher). Although these two approaches seem rather unrelated at first glance, we provide characterisations of different models of Gold-style learning (learning in the limit, conservative inference, and behaviourally correct learning) in terms of query learning. Thus, we describe the circumstances which are necessary to replace limit learners by equally powerful one-shot learners. Our results are valid in the general context of learning indexable classes of recursive languages. This analysis leads to an important observation, namely that there is a natural query learning type hierarchically in-between Gold-style learning in the limit and behaviourally correct learning. Astonishingly, this query learning type can then again be characterised in terms of Gold-style inference.

Keywords: Inductive inference; Query learning; Formal languages; Recursion theory

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1. Introduction

Undeniably, there is no formal scheme spanning all aspects of human learning. Thus each learning model analysed within the scope of learning theory addresses only special facets of our understanding of learning.

For example, Gold's [9] model of *identification in the limit* is concerned with learning as a limiting process of creating, modifying, and improving hypotheses about a target concept. These hypotheses are based upon positive examples[1] for the target concept sequentially offered as information. In the limit, given a gradually growing sequence of positive examples, the learner is supposed to stabilise on a correct guess, but during the learning process in general one will never know whether or not the current hypothesis is already correct. The reason is that at any time in the learning process, the learner has only seen a finite sequence of positive examples and may change its hypothesis upon the next example in the sequence to be presented. Here the ability to change its mind is a crucial feature of the learner.

In contrast to that, Angluin's [2,3] model of *learning with queries* focusses learning as a finite process of interaction between a learner and a teacher. The learner asks questions of a specified type about the target concept and the teacher—having the target concept in mind—answers these questions truthfully. After finitely many steps of interaction the learner is supposed to return its sole hypothesis—correctly describing the target concept. Here the crucial features of the learner are its ability to demand special information on the target concept and its restrictiveness in terms of mind changes. Since a query learner is required to identify the target concept with just a single hypothesis, we refer to this phenomenon as *one-shot learning*.

Note that, in contrast to Gold's model, where the input of the learner consists only of positive examples for the target concept, in query learning the information the learner gets may be of a different quality, so concerning the information resources, the preconditions of query learners differ from those of Gold-style learners. Another difference obviously lies in the constraints on the convergence of the learning process: while a Gold-style learner is only required to converge to a correct guess in the limit, query learners have to deliberately stop the learning process with a single and correct guess.

Our analysis concerns common features and relations between these two seemingly unrelated approaches, thereby focussing our attention on the identification of formal languages, ranging over indexable classes of recursive languages, as target concepts, see Angluin [1], Lange and Zeugmann [13], and Zeugmann and Lange [21]. In this context, our main focus will be on characterisations of Gold-style language learning in terms of learning via queries. Characterising different types of Gold-style language learning in such a way, we will point out interesting correspondences between the two models. Our results illustrate that a difference in the quality of the information resources can be traded for a difference in the requirements of convergence concerning the hypotheses. In particular, it is demonstrated how learners identifying languages in the limit can be replaced by one-shot query learners without any loss of learning power. That means, under certain circumstances the capability of limit learners is equal to that of one-shot learners using queries, or, in other words,

[1] Gold [9] also initiated a model of learning from both positive and negative examples, but we neglect this approach in the sequel.