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On the existence of translations of structured specifications

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A R T I C L E   I N F O

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A B S T R A C T

We provide a set of sufficient conditions for the existence of translations of structured specifications across specification formalisms. The most basic condition is the existence of a translation between the logical systems underlying the specification formalisms, which corresponds to the unstructured situation. Our approach is based upon institution theory and especially upon a recent abstract approach to structured specifications in which both the underlying logics and the structuring systems are treated fully abstractly. Hence our result is applicable to a wide range of actual specification formalisms that may employ different logics as well as different structuring systems, and is very relevant within the context of the fastly developing heterogeneous specification paradigm.

1. Introduction

Formal specification is an important paradigm that assist the development and maintenance of complex software systems; some argue that it is indispensable in the case of critical systems. Complex software systems involve very large specifications that cannot be developed and maintained in the absence of adequate structuring or modularisation. On the other hand, with the recent advance of the heterogeneous specification paradigm, there is a growing interest in the theory of translations between different specification formalisms. While this theory is quite developed at the level of translations between the underlying logics, which corresponds to the unstructured case, very little has been done for the structured specifications level. In my opinion, one reason for that situation lies in the fact until [5] structuring has been always treated concretely by making a choice of a particular set of structuring operators. With that kind of commitment the only variation between specification formalisms that is possible to consider is at the level of the underlying logic. This is a limitation since actual specification systems may involve modularisation constructs that cannot always be traced back to an apriori fixed set of core structuring operators.

The theory of abstractly structured specifications (ASS) of [5] provides a flexible approach to structuring systems and within that framework in [2] the author defines an adequate concept of translation of ASSs that is based upon the concept of comorphism from institution theory [10]. In my opinion, apart from the definition of the concept, the most important contribution of [2] consists of a minimalistic axiomatisation of the concept. In this work we build on the concept of translation of [2] and take a step forward by giving a set of general conditions, widely applicable, for the existence of translations of ASSs.

In the first part of the paper we recall very briefly general concepts of institution theory, and then we recall the main concepts from the theory of ASSs of [5] and also introduce a couple of new concepts required by our work. An important argument presented in the form of an example shows that the semantic normal forms of [5], which play an important role in our main result, is significantly more general than its syntactic counterpart from the literature (e.g. [15]). The final

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technical section starts by recalling the concept of translation of ASSs of [2] and continues with the development of the main result of this paper, namely the existence of translations of ASSs. The general theory is illustrated by a relevant example displaying both concrete logics and concrete sets of structuring operators.

2. Institutions and comorphisms

Institutions [1,9] formalise the intuitive notion of logical system, including the syntax, semantics and the satisfaction between them and have been used intensively in computer science (e.g. [15]) and logic (e.g. [3]).

Definition 2.1 *(Institutions).* An *institution* $\mathcal{I} = (Sign^{\mathcal{I}}, Sen^{\mathcal{I}}, Mod^{\mathcal{I}}, \models^{\mathcal{I}})$ consists of

1. a category $Sign^{\mathcal{I}}$, whose objects are called *signatures*,
2. a functor $Sen^{\mathcal{I}}: Sign^{\mathcal{I}} \to \mathbb{S}\mathrm{et}$ (to the category of sets), giving for each signature a set whose elements are called *sentences* over that signature,
3. a functor $Mod^{\mathcal{I}}: (Sign^{\mathcal{I}})^{\mathrm{op}} \to \mathbb{CAT}$ (from the opposite of $Sign^{\mathcal{I}}$ to the category for categories) giving for each signature Σ a category whose objects are called Σ*-models*, and whose arrows are called Σ-(*model*) *homomorphisms*, and
4. a relation $\models^{\mathcal{I}}_{\Sigma} \subseteq |Mod^{\mathcal{I}}(\Sigma)| \times Sen^{\mathcal{I}}(\Sigma)$ for each $\Sigma \in |Sign^{\mathcal{I}}|$, called Σ*-satisfaction*,

such that for each morphism $\varphi: \Sigma \to \Sigma'$ in $Sign^{\mathcal{I}}$, the *satisfaction condition*

$$M' \models^{\mathcal{I}}_{\Sigma'} Sen^{\mathcal{I}}(\varphi)(\rho) \quad \text{if and only if} \quad Mod^{\mathcal{I}}(\varphi)\big(M'\big) \models^{\mathcal{I}}_{\Sigma} \rho$$

holds for each $M' \in |Mod^{\mathcal{I}}(\Sigma')|$ and $\rho \in Sen^{\mathcal{I}}(\Sigma)$.

We denote the *reduct* functors $Mod^{\mathcal{I}}(\varphi)$ by $\_\restriction_{\varphi}$ and the sentence translations $Sen^{\mathcal{I}}(\varphi)$ by $\varphi(\_)$. When there is no danger of ambiguity, we may skip the superscripts from the notations of the entities of the institution; for example, $Sign^{\mathcal{I}}$ may be denoted simply by *Sign*. An $\mathcal{I}$*-theory* is any pair (Σ, E) such that $\Sigma \in |Sign|$ and $E \subseteq Sen(\Sigma)$. For theory (Σ, E) we let $Mod(\Sigma, E)$ denote the full subcategory of $Mod(\Sigma)$ whose objects are the models satisfying all sentences of E and let $E^{\bullet}$ denote the set of sentences satisfied by all models of $Mod(\Sigma, E)$.

The literature shows myriads of logical systems from computing or from mathematical logic captured as institutions. In fact, an informal thesis underlying institution theory is that any 'logic' based on satisfaction between sentences and models of any kind may be captured by the above definition. Below we recall very briefly a couple of them that we will use in our examples.

Example 2.1 *(Many-sorted First-Order Logic).* In the institution $\mathcal{FOL}$ of many-sorted first-order logic with equality the signatures consist of sorts and typed functions and predicate symbols. The arities of functions are finite strings of sorts. Signature morphisms map symbols such that arities are preserved. Models are first-order structures interpreting sorts as sets, function symbols as functions, and predicate symbols as relations. The sentences are first-order formulas formed from atomic predicate sentences and equations by iteration of logical connectives ($\wedge, \vee, \neg$ etc.) and (first-order) quantifiers $\forall, \exists$. Sentence translation means replacement of the translated symbols. Model reduct means reassembling the model's components according to the signature morphism. Satisfaction is the usual Tarskian satisfaction of a first-order sentence in a first-order structure that is defined by induction on the structure of the sentences. Detailed definitions of variants of this rather common institution, that differ only slightly, can be found in very many places in the literature, e.g. [9,15,3].

Example 2.2 *(Partial Algebra).* The institution $\mathcal{PA}$ of partial algebra is similar to $\mathcal{FOL}$ but functions can also be partial and there are no predicates. Equations evaluate to false if some component term involves some undefinedness or if they evaluate to different values.

Example 2.3 *(Institutions of Theories).* For any institution $\mathcal{I}$ the institution of its theories $\mathcal{I}^{\mathrm{th}}$ has finite theories (Σ, E) (with Σ any $\mathcal{I}$-signature and $E \subseteq Sen^{\mathcal{I}}(\Sigma)$) as signatures. In $\mathcal{I}^{\mathrm{th}}$ the (Σ, E)-sentences are just the Σ-sentences in $\mathcal{I}$ and the (Σ, E)-models are the Σ-models that satisfy all sentences in E. The satisfaction relation of $\mathcal{I}^{\mathrm{th}}$ is inherited from $\mathcal{I}$.

The notion of comorphism [11,16,17,10] represents one of the most important kind of structure preserving mappings between institutions and provides an adequate formalisation of the informal concept of logic translation.

Definition 2.2 *(Comorphisms).* An *institution comorphism* $(\Phi, \alpha, \beta): \mathcal{I} \to \mathcal{I}'$ consists of

1. a functor $\Phi: Sign \to Sign'$,
2. a natural transformation $\alpha: Sen \Rightarrow \Phi; Sen'$, and
3. a natural transformation $\beta: \Phi^{\mathrm{op}}; Mod' \Rightarrow Mod$