Information Processing Letters 115 (2015) 33–39

Contents lists available at ScienceDirect

Information Processing Letters

www.elsevier.com/locate/ipl

Scheduling a variable maintenance and linear deteriorating jobs on a single machine

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ARTICLE INFO

Article history:
Received 16 October 2013
Received in revised form 19 August 2014
Accepted 19 August 2014
Available online 26 August 2014
Communicated by B. Doerr

Keywords:
Scheduling
Computational complexity
Deteriorating jobs
Maintenance

ABSTRACT

We investigate a single machine scheduling problem in which the processing time of a job is a linear function of its starting time and a variable maintenance on the machine must be performed prior to a given deadline. The goals are to minimize the makespan and the total completion time. We prove that both problems are NP-hard. Furthermore, we show that there exists a fully polynomial time approximation scheme for the makespan minimization problem. For the total completion time minimization problem we point out that there exists a fully polynomial time approximation scheme for a special case.

1. Introduction

We consider the following scheduling problem: there is a set of n linear deteriorating jobs $J = \{J_1, J_2, \cdots, J_n\}$ to be non-preemptively processed on a single machine, all of which are available at time 0. For each job J_j, we use p_j and C_j to denote the processing time and the completion time, respectively. The actual processing time p_j of job J_j is defined by $p_j = \alpha_j + \beta_j s_j$, where α_j denotes the basic processing time, β_j denotes the deteriorating rate and s_j denotes the starting time of job J_j, respectively, i.e., p_j is a (general) linear function of s_j. Moreover, a mandatory maintenance must be started before a given deadline s_d on the machine and the duration of the maintenance d is a nonnegative and nondecreasing function of its starting time s (i.e., $d = f(s)$ and $f(s)$ is nonnegative and nondecreasing). Without loss of generality, we also assume that all the data (the basic processing times, the deteriorating rates, the deadline and the duration of maintenance) are integers and the function $f(s)$ can be computed in polynomial time. Furthermore, we assume that jobs cannot fit all together before s_d (i.e., $\alpha_n + \sum_{i=1}^{n}(\alpha_i \prod_{j=i+1}^{n}(1+\beta_j)) > s$) and that every job J_j can be inserted before s_d (i.e., $\alpha_j \le s_d$). The objective is to schedule all the linear deteriorating jobs and determine the starting time of maintenance such that the makespan or the total completion time is minimized.

Let C_{max} $(= \max_j C_j)$ denote the maximum completion time (makespan). Following the three-field notation introduced by Graham et al. [4] and the monograph by Gawiejnowicz [6], we denote our problems by $1, \mathrm{VM}|p_j = \alpha_j + \beta_j s_j|C_{max}$ and $1, \mathrm{VM}|p_j = \alpha_j + \beta_j s_j|\sum_j C_j$ respectively, where "VM" stands for a variable maintenance on the machine.

The above problem is referred to as scheduling deteriorating jobs with machine unavailable constraints and this kind of problem has received considerable attention since 2003 (see, e.g., [14,3,5,9,7,10–12]). For the related monograph, we refer the readers to Gawiejnowicz [6]. All of the models proposed in [14,5,9,7,3,11,12] have the

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http://dx.doi.org/10.1016/j.ipl.2014.08.011

assumptions that the machine starts at time t_0 (>0) (to avoid the trivial case), the processing time of a job is a simple linear function of its starting time (i.e., $p_j = \beta_j s_j$), and the machine unavailable intervals or the duration of maintenance is prefixed, which differ from our model.

Related works Browne and Yechiali [1] first considered the problem of scheduling (general) linear deteriorating jobs (without maintenance) to minimize the makespan. Using the standard interchange principle, they showed that there achieves the optimal makespan according to the nondecreasing order of $\frac{\alpha_j}{\beta_j}$ (i.e. $\frac{\alpha_1}{\beta_1} \le \frac{\alpha_2}{\beta_2} \cdots \le \frac{\alpha_n}{\beta_n}$).

Since 2003, researchers have begun to consider the problems of scheduling simple linear deteriorating jobs with machine unavailable constraints. They assumed that the jobs are available at time t_0 $(=1)$. Depending on whether the job processing can be interrupted by the unavailable intervals or not, there are two versions: resumable version and non-resumable version.

For the resumable version, Wu and Lee [14] first studied the problem of scheduling simple linear deteriorating jobs with an unavailable constraint to minimize the makespan. They showed the model can be solved by using the 0–1 integer programming technique. In some special case, the 0–1 integer programming can be solved in polynomial time. For the general case, the complexity of the problem is open. Gawiejnowicz and Kononov [7] considered the proposed model by Wu and Lee [14]. They proved that the problem is weakly NP-hard and showed that there exists a fully polynomial time approximation scheme. Furthermore, for the problem with two or more unavailable intervals, they showed that there does not exist a polynomial time approximation algorithm with a constant worst-case ratio, unless P = NP. Ji and Cheng [10] considered a single machine scheduling problem in which the processing time of each job is a simple linear deteriorating function of its waiting time with the machine subject to an unavailable constraint. The problem was similar to the proposed problem by Gawiejnowicz and Kononov [7]. For the makespan objective, they showed that the problem can be modeled by 0–1 integer programming and then proved that the problem is weakly NP-hard. Finally they showed that there exists a fully polynomial time approximation scheme. Fan et al. [3] considered the problem of scheduling resumable deteriorating jobs on a single machine with unavailable constraints. The goal is to minimize the total completion time. They proved that the problem with a single unavailable interval is NP-hard and showed that there exists a fully polynomial time approximation scheme. Furthermore, they showed that there does not exist a polynomial time approximation algorithm with a constant worst-case ratio with two or more unavailable intervals, unless P = NP.

For the non-resumable version, Ji et al. [9] considered the single machine scheduling problem in which the jobs are simple linear deteriorating and the machine is subject to an unavailable constraint. The goal is to minimize the makespan or the total completion time. They showed that both problems are weakly NP-hard. Furthermore, for the makespan objective, they proposed an optimal on-line algorithm for the on-line case, and a fully polynomial time approximation scheme for the off-line case. For the total completion time objective, they provided a heuristic algorithm with computational experiments to evaluate its efficiency.

Our results In this paper, we show both problems $1,\mathrm{VM}|p_j = \alpha_j + \beta_j s_j|C_{max}$ and $1,\mathrm{VM}|p_j = \alpha_j + \beta_j s_j|\sum_j C_j$ are NP-hard. Furthermore, for the $1,\mathrm{VM}|p_j = \alpha_j + \beta_j s_j|C_{max}$ problem, we show that there exists a fully polynomial time approximation scheme (FPTAS). For the $1,\mathrm{VM}|p_j = \alpha_j + \beta_j s_j|\sum_j C_j$ problem, we point out that there also exists an FPTAS when $\alpha_j = \beta_j$. But for the general case $\alpha_j \ne \beta_j$ whether it has an FPTAS is open.

The remainder of this paper is organized as follows: In Section 2, we prove that the $1,\mathrm{VM}|p_j = \alpha_j + \beta_j s_j|C_{max}$ problem is weakly NP-hard and show that there exists an FPTAS. In Section 3, we prove that the $1,\mathrm{VM}|p_j = \alpha_j + \beta_j s_j|\sum_j C_j$ problem is also NP-hard and point out that there exists an FPTAS for the special case $\alpha_j = \beta_j$. The concluding remarks are given in Section 4.

2. The $1,\mathrm{VM}|p_j = \alpha_j + \beta_j s_j|C_{max}$ problem

In this section, we consider the $1,\mathrm{VM}|p_j = \alpha_j + \beta_j s_j|C_{max}$ problem. By reducing the Subset Product Problem to the considered problem, we show that the problem is NP-hard and then show it admits an FPTAS.

2.1. Proof of NP-hardness

The NP-hardness proof of the problem is demonstrated by performing a reduction from the NP-complete Subset Product Problem [8].

Subset Product Problem. Given a finite set $S = \{1, 2, \cdots, m\}$, a positive integer $x_j \in Z^+$ for each $j \in S$, and a positive integer B, does there exist a subset $S_1 \subseteq S$ such that the product of the elements in S_1 satisfies $\prod_{j \in S_1} x_j = B$?

Clearly, we can assume $x_j \ge 2$ for all j.

Theorem 1. *The $1,\mathrm{VM}|p_j = \alpha_j + \beta_j s_j|C_{max}$ problem is NP-hard.*

Proof. Given an arbitrary instance I of the Subset Product Problem, we construct an corresponding instance $\hat{I}$ of the $1,\mathrm{VM}|p_j = \alpha_j + \beta_j s_j|C_{max}$ problem as follows:

- There are $n = m$ linear deteriorating jobs such that $\alpha_j = \beta_j = x_j - 1$ for $j = 1, 2, \cdots, n$.
- The starting time of the maintenance s is prior to $B - 1$ (i.e., $s \le s_d = B - 1$) and the duration is B (i.e., $d = f(s) = B$).
- The threshold value G is defined as $G = 2X - 1$, where $X = \prod_{j \in S} x_j$.

Obviously the above construction can be completed in polynomial time. Next, we show that instance I has a solution if and only if there exists a feasible schedule π for instance $\hat{I}$ such that $C_{max}(\pi) \le G$.