Available online at www.sciencedirect.com
SCIENCE DIRECT®
Information Processing Letters 94 (2005) 63–69
Information Processing Letters
www.elsevier.com/locate/ipl

# Scheduling jobs under decreasing linear deterioration

Ji-Bo Wang [a,*], Zun-Quan Xia [b]

[a] *Department of Science, Shenyang Institute of Aeronautical Engineering, Shenyang 110034, People's Republic of China*
[b] *Department of Applied Mathematics, Dalian University of Technology, Dalian 116024, People's Republic of China*

Received 21 April 2004; received in revised form 14 December 2004

Communicated by M. Yamashita

**Abstract**

This paper considers the scheduling problems under decreasing linear deterioration. Deterioration of a job means that its processing time is a function of its execution start time. Optimal algorithms are presented respectively for single machine scheduling of minimizing the makespan, maximum lateness, maximum cost and number of late jobs. For two-machine flow shop scheduling problem to minimize the makespan, it is proved that the optimal schedule can be obtained by Johnson's rule. If the processing times of operations are equal for each job, flow shop scheduling problems can be transformed into single machine scheduling problems.


*Keywords:* Scheduling; Single machine; Flow shop; Linear deterioration

## 1. Introduction

In the classical scheduling theory, job processing times are considered to be constant. In practice, however, we often encounter settings in which processing time increases or decreases over time. Researchers have formulated this phenomenon into different models and solved different problems for various criteria. An extensive survey of different models and problems concerning start time dependent job processing times can be found in [1,7]. Generally, there are two types of models describing this kind of processes. The first type is devoted to the problems in which the job processing time is characterized by a non-decreasing function, and the second type concerns problems in which the job processing time is given by a non-increasing function. Applications of these models can be found, among others, in fire fighting, emergency medicine, police, machine maintenance, and computer science.

Some common examples of the problem in which the job processing time is an increasing start time dependent function can be found in the areas of scheduling maintenance, cleaning assignments or metallurgy, in which any delay often implies additional effort (or time) to accomplish the job. On the other hand, an ex-

* Corresponding author.
*E-mail address:* wangjibo75@yahoo.com.cn (J.-B. Wang).

0020-0190/$ – see front matter © 2005 Published by Elsevier B.V.
doi:10.1016/j.ipl.2004.12.018

ample considering the so-called “learning effect” can be described by a non-increasing start time dependent function. Assume that a worker has to assemble a large number of similar products. The time required by the worker to assemble one product depends on his knowledge, skills, organization of his working place and others. The worker learns how to produce. After some time, he is better skilled, his working place is better organized and his knowledge has increased. As a result of his learning, the time required to assemble one product decreases. Another example is the process by which aerial threats are to be recognized by a radar station [9,15]. In this case, a radar station has detected some objects approaching it. The time required to recognize the objects decreases as the objects get closer. Thus, the later the objects are detected, the less time needed for their recognition.

Browne and Yechiali [4] consider a scheduling problem in which the processing times of the jobs are not constant over time. $n$ jobs have to be processed on a single machine to minimize the makespan. Job $J_i$ is characterized by: (1) a “basic” processing time $a_i$, the length of time required to complete the job if it were scheduled first, i.e., at $t = 0$, and (2) a parameter $b_i$ that jointly with $a_i$ determines the job’s (actual) processing time at $t > 0$; $b_i$ can be interpreted as the growth rate of the processing time of job $J_i$. Assuming linear deterioration, i.e., the processing time of the job increases linearly with its starting time $t$, the actual processing time is $p_i(t) = a_i + b_i t$. This problem can be solved optimally by scheduling jobs in an increasing order of $a_i/b_i$, the ratio of the basic processing time to the growth rate. Mosheiov [13] considers the problem that all jobs are characterized by a common positive basic processing time. Using this basic assumption, Mosheiov proves that the optimal schedule to minimize flowtime is symmetric and has a V-shaped property with respect to the increasing rates. Mosheiov [14] considers the following objective functions: makespan, total flow time, total weighted completion time, total lateness, maximum lateness and maximum tardiness, and number of tardy jobs. When the values of the normal processing time equal zero, i.e., $p_i(t) = b_i t$, all these problems can be solved polynomially. Bachman and Janiak [3] consider other variants of the linear model of start time dependent job processing times. They assume that for all jobs, the increasing rates are $k$ times greater than the basic processing times $a_j$, i.e., $b_j = ka_j$, $k > 0$. Cheng and Ding [6] consider the scheduling model in which each job has a normal processing time which deteriorates as a step function if its starting time is beyond a given deterioration rate. They show that the flow time problem with identical task deteriorating dates is NP-complete and suggest a pseudo-polynomial algorithm for the makespan problem. They also introduce a general method of solution for the flow time problems. Zhao et al. [18] consider a special type of the actual processing time, which is $p_i(t) = a_i(a + bt)$, where $a$ and $b$ are positive constants. They prove that the single machine scheduling problems of minimizing makespan, sum of weighted completion times, maximum lateness and maximum cost is polynomially solvable, and the two-machine flow shop scheduling to minimize the makespan, for this case the optimal solution can be obtained by Johnson’ rule.

Apart from the increasing linear model for the job processing times, there is also a decreasing linear model. This model is introduced by Ho et al. [9]. It also contains two parts, but in contrast to the increasing model, the start time dependent part is described by a decreasing rate. Ho et al. [9] consider the problem of solution feasibility with deadline restrictions. Cheng and Ding [5] consider some problems with a decreasing linear model of the job processing times, but with ready time and deadline restrictions. They identify some interesting relationships between the linear models with decreasing and increasing start time dependent parts. Ng et al. [15] also consider three scheduling problems with a decreasing linear model of the job processing times, where the objective function is to minimize the total completion time, and two of the problems are solved optimally. A pseudopolynomial time algorithm is constructed to solve the third problem using dynamic programming. Some interesting relationships between the linear model with decreasing and increasing start time dependent parts have also been presented by Ng et al. [15]. Bachman et al. [2] consider the single machine scheduling problem with start time dependent job processing times. They prove that the problem of minimizing the total weighted completion time is NP-hard. They also consider some special cases.

In this paper, we consider the scheduling problems with a special type of deteriorating jobs in which the processing time of each job is given by a decreasing