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Axial solutions for multiple objective linear problems. An application to target setting in DEA models with preferences

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ABSTRACT

In this paper we introduce the class of axial solutions for multiple objective optimization problems in contexts in which partial information on preference weights is available. These solutions combine the use of an improvement axis to direct the search of the most preferred result with the concept of efficiency with respect to preference information.

In addition, we show how this solution is used in data envelopment analysis in order to set realistic targets in accordance with either the partially specified preferences of a single decision maker, or the different individual preference information provided by the members of a group of decision makers.

1. Introduction

The difficulty of representation of the underlying preferences of the decision maker (DM) and the elicitation of the weights of importance of the objectives are central issues in multiple objective decision-making and, as such, have been extensively treated in multiple objective programming literature.

Here we present an approach which partly deals with these difficulties. We consider an informational environment that allows imprecise information on the objectives weights, that is, the values of these weights are not precisely stated, with only partial information about them being available. In this context we assume that preferences are originally additive, however, an additional element is included to enrich the representation in the form of an improvement axis, which influences the selection of weights in each particular problem.

Previous work on the treatment of multi-objective decision problems with partial information has focused on the case where the preferences of the DM are represented by additive value functions and the main goal has been the reduction of the Pareto-optimal set of points to those consistent with the information available (see, for instance, [23,18]). There is also a relevant stream of related literature devoted to the analysis of multi-objective linear problems, in which objective function coefficients or preferential weight coefficients are not exactly specified, but given as intervals or by means of linear relations (see, for instance, [17,14,24,25]). Whereas these papers focus mainly on the sensitivity of a given solution to feasible changes in the parameters, our present research is devoted to the proposal of specific solutions for multiple objective decision problems and to their potential applications.

Recent work on solutions to multiple objective problems in the context of partial information can be found in Hinojosa and Mármol [15]. In that paper, the classic utilitarian solution, which relies on an additive preference structure, is extended to this more general framework, whereby the class of *inequality averse utilitarian solutions* is introduced and investigated. These are solutions which exhibit a property of neutrality with respect to the objectives, resulting in equal treatment of the objectives which are not differentiated in the preference information available.

With the introduction of the class of *axial solutions* in the present paper, we go a step further in the definition of solution concepts based on a utilitarian notion. When using axial solutions, a weighted sum of the objectives also directs the search for the results. However, as a consequence of the inclusion of an improvement axis, the different objectives are not treated equally even when the preference information does not discriminate between them. A relevant result here, which permits the effective computation of the solutions, is the characterization of those results obtained with axial solutions to multiple objective linear problems (MOLPs) as the solutions to single-objective linear problems.

The analysis presented herein can be interpreted in terms of group decision making. When a group of DMs has to solve a multiple objective problem but do not agree on the importance of the criteria, each member of the group will provide a different vector of weights for the objectives and a decision has to be made by taking into account all the information. The lack of consensus in these situations can be represented by this model of partial

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information on the weights of the objectives where the preference structure is induced by the vectors of individual assessments. The axial solution in this case provides a result which takes the improvement direction into account in such a way that it cannot be improved simultaneously with respect to the preferences of all the DMs.

The second part of the paper is devoted to the investigation of how axial solutions can be used in data envelopment analysis (DEA).

The underlying assumption in DEA is that no output or input is more important than another, although, in practice, there generally exists a DM or several DMs who have preferences over the outputs or inputs involved. We want to investigate those situations in which preference information has to be taken into account in order to analyze the efficiency of the decision-making units and to select a realistic target in accordance with these preferences.

DEA has been a growing area of interest in management science as reflected in the many extensions of the original work of Charnes et al. [5] which have been proposed and used successfully in a wide range of applications. Many of the underlying ideas and concepts of these new developments, and also of the original models, are rooted in multiple objective techniques and approaches. The links between multiple objective linear programming (MOLP) and data envelopment analysis (DEA) have been the object of attention in recent literature [20,21,12,13,8,26]. Examples also exist showing how DEA methodologies can be used as an aid to analyze multiple objective problems [9,1,2].

The present paper also makes a contribution to the investigation of how concepts originally proposed in multiple objective contexts can prove useful in dealing with and interpreting DEA models. Therefore, the goal of the paper is two-fold. Firstly, we present a new class of solutions to multiple objective optimization problems which incorporates partial preference information: axial solutions. We then show how the notion of axial solution can be used for target setting in DEA models with preferences, thereby producing significant advantages.

Several procedures have been proposed and used to incorporate preference information in order to improve the discrimination of the analysis and to set realistic targets for inefficient DMUs. Some methods incorporate preference information in the form of hypothetical DMUs and/or by estimating an unknown value function [11,12,26].

Another category of methods are those which deal with the reduction of the flexibility of the weights involved in DEA models. These approaches include the proposals of Dyson and Thanassoulis [10], the "assurance-region" approach [22], and the "cone-ratio" method to incorporate restrictions in envelopment DEA models proposed in Charnes et al. [6]. These methods produce a reduction in the production possibility set and have the drawback that the efficient radial targets obtained for the inefficient DMUs are not generally technologically feasible. Podinovski [19] presents a related method based on the incorporation of realistic production trade-offs in the models in such a way that the efficiency measure retains its meaning as a radial improvement factor.

The analysis that we present here can be located in this second category of methods since restriction of preferential weights is involved. We consider a preference structure in the multiple objective linear model associated with the envelopment model. Partial information on preferences can be provided by means of linear relations between the importance weights that a single decision maker attaches to the outputs. Alternatively, each member of a group might provide a different vector of weights. The information is incorporated into the definition of efficiency, and when the problem to obtain the axial solution for the corresponding MOLP is formally stated, a generalized envelopment DEA model is obtained. The resulting model is a particular case of the cone-ratio model [6] when only output weights are restricted. As a consequence, our analysis of axial solutions provides further insight on the cone-ratio approach and of its geometry. In fact, it offers an interpretation of the efficiency measure obtained in this approach.

The results that link axial solutions with data envelopment analysis are presented here in a context where constant returns to scale are assumed thereby yielding a generalization of the CCR model [5]. However, the same ideas can be applied with variable returns to scale resulting in an extension of the BCC model [3] which accommodates partial preferential information. The potential of these new models to deal with various applications will be the object of further research.

The paper is structured as follows. Section 2 is devoted to the representation of preference structures by means of partial information and the corresponding concepts of efficiency. The idea of improvement axis is also presented in this section. In Section 3, we introduce the class of axial solutions for multiple objective decision problems. We characterize these solutions as the optimal solutions of certain linear single-objective problems. In Section 4, the concept of axial solution is applied in the analysis of DEA models with partial information, and a generalized DEA model specially designed to accommodate preference information is proposed. The last section contains an illustration in order to show how our approach is used to analyze the efficiency of a group of banks under different preference situations.

2. Partial preference information and improvement axis

Suppose an optimization problem has s objectives reflecting the different purposes of a DM. Such a problem will be denoted by (Ω,f) and can be represented in a general form as follows:

$$\begin{aligned} \max \quad & f(\lambda) = [f_1(\lambda), \ldots, f_r(\lambda), \ldots, f_s(\lambda)] \\ \text{s.t.} \quad & \lambda \in \Omega, \end{aligned}$$

where Ω is the feasible set in the decision space,[1] $\Omega \subseteq \mathbb{R}^n$, and f is a mapping, $f : \mathbb{R}^n \to \mathbb{R}^s$, where f_r, $r=1,2,\ldots,s$, are the continuously differentiable objective functions. In this paper we will consider multiple objective linear problems, therefore, f_r are linear functions of λ, and Ω is a polyhedron. The feasible set in the objective space, $f(\Omega) = \{z \in \mathbb{R}^s | z = f(\lambda), \lambda \in \Omega\}$, is then also a polyhedron.

Any vector $\lambda \in \Omega$ is called a *feasible solution* of the problem (Ω,f). Among them, the *most preferred solutions* can be found by requiring some properties to be fulfilled.

Pareto-optimality is a primary requirement for the solutions to multiple-objective problems.[2] A feasible solution $\lambda^* \in \Omega$ is said to be *Pareto-optimal* or *efficient* if there is no feasible solution $\lambda \in \Omega$ such that $f_r(\lambda) > f_r(\lambda^*)$ for all $r=1,\ldots,s$. If there is no other feasible solution, $\lambda \in \Omega$, such that $f_r(\lambda) \geq f_r(\lambda^*)$ for all $r=1,\ldots,s$ and $f(\lambda^*) \neq f(\lambda)$, then $\lambda^* \in \Omega$ is *strongly Pareto-optimal* or *strongly efficient*. If $f_r(\lambda^*) \geq f_r(\lambda)$ for all $r=1,\ldots,s$ and for all $\lambda \in \Omega$, then $\lambda^* \in \Omega$ is an *ideal solution*.

The set of all Pareto-optimal (strongly Pareto-optimal) points for (Ω,f) is denoted by $E(\Omega,f)$ $(SE(\Omega,f))$. In most MOLPs, the sets $E(\Omega,f)$ and $SE(\Omega,f)$ contain too many points for the DM to easily

[1] Let $\mathbb{R}$ $(\mathbb{R}_+,\mathbb{R}_{++})$ denote the set of all (non-negative, positive) real numbers and $\mathbb{R}^n$ $(\mathbb{R}^n_+,\mathbb{R}^n_{++})$ the n-fold Cartesian product of $\mathbb{R}$ $(\mathbb{R}_+,\mathbb{R}_{++})$.

[2] The terms Pareto-optimal and efficient are used here to refer to the solutions in the decision space, Ω. However, when no confusion is possible, we will also use the term efficient to refer to the corresponding points in the space of objectives.