Information Processing Letters 115 (2015) 87–92

Contents lists available at ScienceDirect

Information Processing Letters

www.elsevier.com/locate/ipl

Completeness of context-sensitive rewriting ☆

Salvador Lucas [a,b,*]

[a] CS Dept. at the University of Illinois at Urbana-Champaign, USA
[b] DSIC, Universitat Politècnica de València, Spain

ARTICLE INFO

Article history:
Received 17 March 2014
Received in revised form 7 July 2014
Accepted 8 July 2014
Available online 15 July 2014
Communicated by J.L. Fiadeiro

Keywords:
Algebraic semantics
Context-sensitive rewriting
Formal reasoning
Programming languages

ABSTRACT

Restrictions of rewriting may turn normal forms of some terms *unreachable*, leading to *incomplete* computations. *Context-sensitive rewriting* (CSR) is the restriction of rewriting that only permits reductions on arguments selected by a *replacement map* μ, which associates a subset $\mu(f)$ of argument indices with each function symbol f. Hendrix and Meseguer defined an *algebraic semantics* for Term Rewriting Systems (TRSs) executing CSR that can be used to reason about programs written in programming languages like CafeOBJ and Maude, where such replacement restrictions can be specified in programs. Semantic completeness of CSR was also defined. In this paper we show that *canonical replacement maps*, which play a prominent role in simulating rewriting computations with CSR, are *necessary* for completeness in important classes of TRSs.

© 2014 Elsevier B.V. All rights reserved.

1. Introduction

Recursive definitions that use conditional expressions often require the use of syntactic restrictions to obtain *terminating* programs.

Example 1. The following TRS encodes a definition of the *factorial* function:

$$\mathsf{p}(\mathsf{s}(x)) \to x$$

$$0 + x \to x$$

$$\mathsf{s}(x) + y \to \mathsf{s}(x + y)$$

$$\mathsf{zero}(0) \to \mathsf{true}$$

$$0 \times y \to 0$$

$$s(x) \times y \to y + (x \times y)$$

$$\mathsf{zero}(\mathsf{s}(x)) \to \mathsf{false}$$

$$\mathsf{if}(\mathsf{true}, x, y) \to x$$

$$\mathsf{if}(\mathsf{false}, x, y) \to y$$

$$\mathsf{fact}(x) \to \mathsf{if}\big(\mathsf{zero}(x), \mathsf{s}(0), \mathsf{fact}\big(\mathsf{p}(x)\big) \times x\big)$$

Without any restriction on the evaluation of the arguments of if, the last rule makes the program *nonterminating*. Most implementations first (or just) evaluate the boolean condition and *restrict* the evaluation of the other arguments.

In *context-sensitive rewriting* (CSR [4,5]), fixed restrictions on reductions are imposed by means of a *replacement map* μ that, for each k-ary symbol f, specifies the argument positions $i \in \mu(f) \subseteq \{1, \ldots, k\}$ which *can* be rewritten. We say that a replacement map is *compatible* with a given rule $\ell \to r$ of a TRS $\mathcal{R}$, if the positions of nonvariable symbols in ℓ are always *reducible* under μ. We say that μ is a *canonical* replacement map if it is compatible with *all rules* of the TRS $\mathcal{R}$. The use of canonical replacement maps μ ensures that context-sensitive computations may stop yielding head-normal forms, values or even normal forms [4,5]. With $\mu(\mathsf{if}) = \{1\}$ we obtain a *terminating behavior* for

☆ Developed during a sabbatical year at UIUC. Supported by NSF CNS 13-19109, MINECO project TIN2010-21062-C02-02, GV (Generalitat Valenciana) Grants BEST/2014/026 and PROMETEO/2011/052.

* Correspondence to: DSIC, Universitat Politècnica de València, Camino de Vera s/n, 46022 Valencia, Spain.

http://dx.doi.org/10.1016/j.ipl.2014.07.004
0020-0190/© 2014 Elsevier B.V. All rights reserved.

$\mathcal{R}$ in Example 1 which can be proved with existing termination tools like MU-TERM. Indeed, CSR can compute the value $s^{n!}(0)$ of any call $\mathsf{fact}(s^n(0))$, for $n \geq 0$, without running in any termination problem (Example 4). In contrast, a *normalizing* evaluation strategy (e.g., the leftmost–outermost rewriting strategy, which is normalizing for $\mathcal{R}$) does not stop with terms like $\mathsf{fact}(\mathsf{p}(0))$ having no normal form.

Recently, several authors have investigated semantic properties of computations with CSR [3,6]. The motivation is devising appropriate models to (inductively) reason about properties of programs of programming languages like CafeOBJ [2] or Maude [1], where the specification of context-sensitive replacement restrictions is allowed. Hendrix and Meseguer introduce a number of semantic properties (μ*-canonical completeness*, μ*-semantic completeness* and μ*-sufficient completeness*) that can be used to guarantee that CSR is well-suited to tackle the desired formal framework for reasoning about programs with replacement restrictions. The results in [3] are completely general and do not refer to any specific class of TRSs or replacement maps. This is in sharp contrast with the analysis of completeness of CSR in [4,5], where *left linearity* of TRSs and *canonicity* of replacement maps are required. In this paper we show that, for orthogonal TRSs, the use of a canonical replacement map is *necessary* for enjoying the three previous semantic properties. For canonical completeness, it is also necessary that *all arguments of all constructor symbols be* μ*-replacing* (not only with orthogonal TRSs but with any TRS). Furthermore, being completely defined (i.e., ground normal forms contain no defined symbols) is also *necessary* for TRSs $\mathcal{R}$ that enjoy the considered semantic properties.

2. Preliminaries

Given a set A, a binary relation $\mathsf{R} \subseteq A \times A$ is *terminating* if there is no infinite sequence $a_1, a_2, \ldots, a_n, \ldots$ such that for all $i \geq 1$, $a_i \in A$ and $a_i \,\mathsf{R}\, a_{i+1}$. In this paper, $\mathcal{X}$ denotes a countable set of variables and $\mathcal{F}$ denotes a signature, i.e., a set of function symbols $\{f, g, \ldots\}$, each having a fixed arity given by a mapping $ar : \mathcal{F} \to \mathbb{N}$. The set of terms built from $\mathcal{F}$ and $\mathcal{X}$ is $\mathcal{T}(\mathcal{F}, \mathcal{X})$. A term without variables is called *ground*. The set of ground terms is $\mathcal{T}(\mathcal{F})$. A term is said to be linear if it has no multiple occurrences of a single variable. Terms are viewed as labelled trees in the usual way. Positions $p, q, \ldots$ are represented by chains of positive natural numbers used to address subterms of t. We denote the empty chain by Λ. Given positions p, q, we denote its concatenation as $p.q$. Positions are ordered by the standard prefix ordering $\leq$. Given a set of positions P, $minimal_{\leq}(P)$ is the set of minimal positions of P (ordered by $\leq$). If p is a position, and Q is a set of positions, $p.Q = \{p.q \mid q \in Q\}$. The set of positions of a term t is $\mathcal{P}os(t)$. Positions of non-variable symbols in t are denoted as $\mathcal{P}os_{\mathcal{F}}(t)$, and $\mathcal{P}os_{\mathcal{X}}(t)$ are the positions of variables. The subterm of t at position p is denoted as $t|_p$ and $t[s]_p$ is t with $t|_p$ replaced by s. The symbol labelling the root of t is denoted as $root(t)$. A rewrite rule is an ordered pair (ℓ, r), written $\ell \to r$ (or $\alpha : \ell \to r$ if labelled α for further reference), with $\ell, r \in \mathcal{T}(\mathcal{F}, \mathcal{X})$, $l \notin \mathcal{X}$ and $\mathcal{V}ar(r) \subseteq \mathcal{V}ar(l)$. The left-hand side (*lhs*) of the rule is ℓ and r is the right-hand side (*rhs*). A TRS is a pair $\mathcal{R} = (\mathcal{F}, R)$ where R is a set of rewrite rules. $L(\mathcal{R})$ denotes the set of *lhs*'s of $\mathcal{R}$. An instance $\sigma(l)$ of a *lhs* l of a rule is a redex. The set of redex positions in t is $\mathcal{P}os_{\mathcal{R}}(t)$. If $\mathcal{P}os_{\mathcal{R}}(t) = \varnothing$, then t is a *normal form*. Let $\mathsf{NF}_{\mathcal{R}}$ (resp. $\mathsf{GNF}_{\mathcal{R}}$) be the set of (ground) normal forms of $\mathcal{R}$. A TRS $\mathcal{R}$ is left-linear if for all $l \in L(\mathcal{R})$, l is a linear term. Given $\mathcal{R} = (\mathcal{F}, R)$, we consider $\mathcal{F}$ as the disjoint union $\mathcal{F} = \mathcal{C} \uplus \mathcal{D}$ of symbols $c \in \mathcal{C}$, called *constructors* and symbols $f \in \mathcal{D}$, called *defined functions*, where $\mathcal{D} = \{root(l) \mid l \to r \in R\}$ and $\mathcal{C} = \mathcal{F} - \mathcal{D}$. Then, $\mathcal{T}(\mathcal{C}, \mathcal{X})$ (resp. $\mathcal{T}(\mathcal{C})$) is the set of (ground) constructor terms. A defined symbol f is *completely defined* if there is no $t \in \mathsf{GNF}_{\mathcal{R}}$ such that $root(t) = f$. A TRS $\mathcal{R} = (\mathcal{C} \uplus \mathcal{D}, R)$ is a *constructor system* (CS) if for all $f(\ell_1, \ldots, \ell_k) \to r \in R$, $\ell_i \in \mathcal{T}(\mathcal{C}, \mathcal{X})$, for $1 \leq i \leq k$. A term $t \in \mathcal{T}(\mathcal{F}, \mathcal{X})$ rewrites (in one-step) to s (at position p), written $t \xrightarrow{p}_{\mathcal{R}} s$ (or just $t \to s$), if $t|_p = \sigma(\ell)$ and $s = t[\sigma(r)]_p$, for some rule $\ell \to r \in R$, $p \in \mathcal{P}os(t)$ and substitution σ. We say that s rewrites to t if $s \to^* t$. A TRS is terminating if $\to$ is terminating. A term is said to be normalizing if it rewrites into a normal form. A TRS $\mathcal{R}$ is normalizing if every term is normalizing. A TRS is called *completely defined* or *exhaustive* if no ground normal form contains a defined symbol.

Context-sensitive rewriting A mapping $\mu : \mathcal{F} \to \wp(\mathbb{N})$ is a *replacement map* ($\mathcal{F}$-map) if for all $f \in \mathcal{F}$, $\mu(f) \subseteq \{1, \ldots, ar(f)\}$ [4]. $M_{\mathcal{F}}$ is the set of $\mathcal{F}$-maps. For a TRS $\mathcal{R} = (\mathcal{F}, R)$, we use $M_{\mathcal{R}}$ instead of $M_{\mathcal{F}}$. We write $\mu \sqsubseteq \mu'$ if for all $f \in \mathcal{F}$, $\mu(f) \subseteq \mu'(f)$ and say that μ is *more restrictive* than μ'. We write $(\mu \sqcup \mu')(f) = \mu(f) \cup \mu'(f)$ for all $f \in \mathcal{F}$. The set of μ*-replacing positions* of t is: $\mathcal{P}os^{\mu}(t) = \{\Lambda\}$, if $t \in \mathcal{X}$ and $\mathcal{P}os^{\mu}(t) = \{\Lambda\} \cup \bigcup_{i \in \mu(root(t))} i.\mathcal{P}os^{\mu}(t|_i)$ if $t \notin \mathcal{X}$. The set of *non-*μ*-replacing* positions is $\overline{\mathcal{P}os^{\mu}}(t) = \mathcal{P}os(t) - \mathcal{P}os^{\mu}(t)$. The non-$\mu$-replacing positions of t have a *frontier* set $\mathcal{F}r^{\mu}(t) = minimal_{\leq}(\overline{\mathcal{P}os^{\mu}}(t))$ with the *active* positions. The *maximal replacing context* $MRC^{\mu}(t) = t[\Box]_{\mathcal{F}r^{\mu}(t)}$ of t is the part of t whose positions are μ-replacing in t [5]. The *canonical replacement map* $\mu^{can}_{\mathcal{R}}$ of $\mathcal{R}$ is *the most restrictive replacement map ensuring that the nonvariable subterms of the left-hand sides of the rules of* $\mathcal{R}$ *are* μ*-replacing* [4,5]: for each $f \in \mathcal{F}$ and $i \in \{1, \ldots, ar(f)\}$, $i \in \mu^{can}_{\mathcal{R}}(f)$ iff $\exists \ell \in L(\mathcal{R})$, $p \in \mathcal{P}os_{\mathcal{F}}(l)$, $(root(l|_p) = f \wedge p.i \in \mathcal{P}os_{\mathcal{F}}(l))$. Given a TRS $\mathcal{R}$, $CM_{\mathcal{R}} = \{\mu \in M_{\mathcal{R}} \mid \mu^{can}_{\mathcal{R}} \sqsubseteq \mu\}$ is the set of replacement maps that are equal to or *less restrictive* than the canonical replacement map. If $\mu \in CM_{\mathcal{R}}$, we say that μ is *a* canonical replacement map for $\mathcal{R}$. Given a TRS $\mathcal{R} = (\mathcal{F}, R)$, $\mu \in M_{\mathcal{R}}$, and $s, t \in \mathcal{T}(\mathcal{F}, \mathcal{X})$, s μ-rewrites to t, written $s \stackrel{p}{\hookrightarrow}_{\mathcal{R},\mu} t$ (or $s \hookrightarrow_{\mathcal{R},\mu} t$, $s \hookrightarrow_{\mu} t$, or even $s \hookrightarrow t$), if $s \xrightarrow{p}_{\mathcal{R}} t$ and $p \in \mathcal{P}os^{\mu}(s)$ [4]. If $\mu \in CM_{\mathcal{R}}$, we often say that $\hookrightarrow_{\mu}$ performs *canonical* context-sensitive rewriting steps [5]. A term t without replacing redexes (i.e., $\mathcal{P}os^{\mu}_{\mathcal{R}}(t) = \varnothing$) is called a μ*-normal form*, and $\mathsf{NF}^{\mu}_{\mathcal{R}}$ (resp. $\mathsf{GNF}^{\mu}_{\mathcal{R}}$) is the set of (ground) μ-normal forms of $\mathcal{R}$. We write $s \hookrightarrow^! t$ if t is a μ-normal form of s, i.e., $s \hookrightarrow^* t$ and $t \in \mathsf{NF}^{\mu}_{\mathcal{R}}$; if $s \hookrightarrow^! t$ and $s \hookrightarrow^! t'$ imply $t = t'$, then we denote such *unique* μ-normal form of s as $s{\downarrow_{\mu}}$.

Download English Version:

https://daneshyari.com/en/article/10331864

Download Persian Version:

https://daneshyari.com/article/10331864

Daneshyari.com