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The hamiltonicity of generalized honeycomb torus networks

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ABSTRACT

Yang et al. (2004) [8] proved that the generalized honeycomb torus $GHT(m,n,d)$ is hamiltonian, but their proofs are not sufficient when the width m is odd. In this paper, we propose a series of procedures for constructing hamiltonian cycles in generalized honeycomb tori, which apply to every instance of $GHT(m,n,d)$ with odd width m.

1. Introduction

The advent of very large scale integrated circuit technology has enabled the construction of very complex and large parallel computing systems. By most accounts, a supercomputer achieves its gains by increasing the number of processing elements, rather than by using faster processors. The most difficult technical problem in constructing a supercomputer will be the design of the interconnection network through which the processors communicate and exchange data with each other. Therefore, selecting an appropriate and adequate topological structure of interconnection networks will become a critical issue in the field of parallel computing [6].

There exist a lot of mutually conflicting requirements in designing the topology of an interconnection network, such that it is almost impossible to design a network which is optimal from all aspects. One has to design a suitable network according to the requirements and its properties. Many efficient algorithms were originally designed based on rings for solving a variety of algebraic problems, graph problems and some parallel applications, such as those in image and signal processing. Thus, it is important to have an effective cycle in a network, preferably the hamiltonian cycle.

Stojmenovic [7] introduced three classes of honeycomb torus networks: hexagonal honeycomb torus, rectangular honeycomb torus and parallelogramic honeycomb torus. Megson et al. [4,5] proved that a hexagonal honeycomb torus is hamiltonian and fault-tolerant hamiltonian with two adjacent faulty vertices. Cho and Hsu [1] proposed the generalized honeycomb torus, which includes the above mentioned honeycomb tori as special instances. Yang et al. [8] proved that all generalized honeycomb tori are hamiltonian. However, we found that their proofs are not sufficient when the width m is odd.

In this paper, we propose a series of procedures for constructing the hamiltonian cycles in generalized honeycomb tori, which apply to every instance of $GHT(m,n,d)$ with odd width m. The rest of this paper is organized as follows. Section 2 gives definitions and notations. Section 3 presents the main result of the paper. Section 4 makes the concluding remarks.

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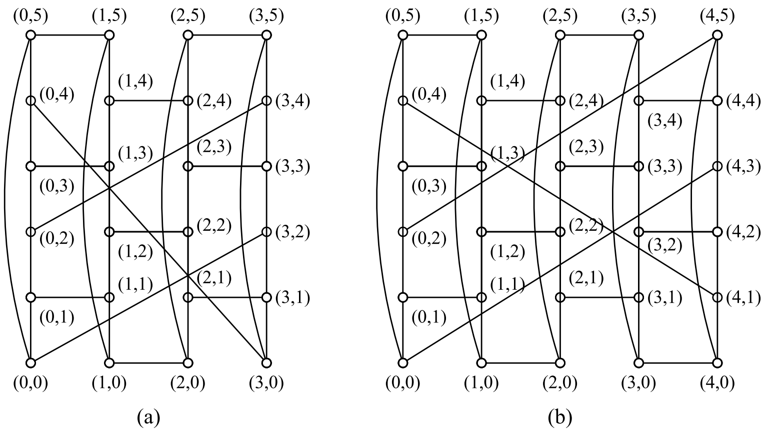

Fig. 1. Two examples of generalized honeycomb tori. (a) $GHT(4,6,2)$. (b) $GHT(5,6,3)$.

2. Definitions and notations

The topological structure of an interconnection network can be modeled by a graph $G=(V,E)$, where vertices and edges correspond to processors and communication links between processors, respectively. This fact has been universally accepted and used by computer scientists and engineers. Moreover, practically it has been demonstrated that graph theory is a fundamental and powerful mathematical tool for designing and analyzing topological structure of interconnection networks.

A hamiltonian cycle of a graph is a cycle that traverses every vertex of the graph exactly once. A graph is hamiltonian if it contains a hamiltonian cycle. We follow [2] for graph-theoretical terminology and notations not defined here.

Definition 2.1. (See [1].) Let n be a positive even integer, $m \geq 2$ be a positive integer, and d be a nonnegative integer which is less than n and of the same parity with m. An (m,n,d) generalized honeycomb torus, denoted by $GHT(m,n,d)$, is a graph with the vertex set

$$V\big(GHT(m,n,d)\big)=\big\{(i,j): 0 \leq i \leq m-1, 0 \leq j \leq n-1\big\}.$$

m, n and d are named the width, height and slope of $GHT(m,n,d)$. For a vertex (i,j) of $GHT(m,n,d)$, i and j are called its first and the second component, respectively. Here and in what follows, all arithmetic operations carried out on the first and second components are modulo m and n, respectively. Two vertices (x_1,y_1) and (x_2,y_2) with $x_1 \leq x_2$ are adjacent if and only if one of the follow conditions is satisfied:

(1) $(x_2,y_2)=(x_1,y_1+1)$ or $(x_2,y_2)=(x_1,y_1-1)$;
(2) $0 \leq x_1 \leq m-2$, x_1+y_1 is odd, and $(x_2,y_2)=(x_1+1,y_1)$;
(3) $x_1=0$, y_1 is even, and $(x_2,y_2)=(m-1,y_1+d)$.

Fig. 1 gives two examples of generalized honeycomb tori. We can easily see that generalized honeycomb tori are 3-regular bipartite graphs.

Yang et al. [8] proved that every generalized honeycomb torus is hamiltonian. However, when m is odd, their scheme for constructing hamiltonian cycles is only valid when $gcd(\frac{2n}{gcd(n,d+1)},\frac{2(d+1)}{gcd(n,d+1)-1})=1$. For example in $GHT(5,12,7)$, they get three vertex-disjoint cycles instead of a hamiltonian cycle (see Fig. 2).

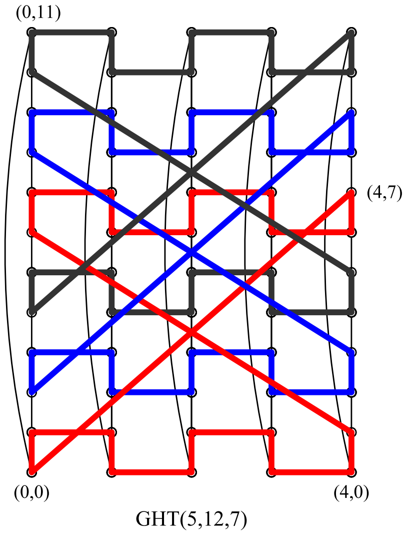

Fig. 2. Three vertex-disjoint cycles in $GHT(5,12,7)$.

Definition 2.2. Given two positive integers a and b where $a \geq b$, $GC(a,b)$ is a graph defined by the vertex set $\{0,1,\dots,a-1\}$ and the edge set $\{(i,i+b): 0 \leq i \leq a-1\}$, where the arithmetic is modulo a [8].

Given two positive integers a and b where $a \geq b$, let $gcd(a,b)$ denote the greatest common divisor of a and b. The following lemmas will be useful in this paper.

Lemma 2.1. *If $gcd(a,b)=1$, then $GC(a,b)$ is a cycle [8].*

Lemma 2.2. *If $gcd(a,b)=c \geq 2$, then $GC(a,b)$ is composed of c vertex-disjoint cycles $\langle 0,b,2b,3b,\dots,(a/c-1)b,0\rangle$, $\langle 1,b+1,2b+1,3b+1,\dots,(a/c-1)b+1,1\rangle$, $\langle 2,b+2,2b+2,3b+2,\dots,(a/c-1)b+2,2\rangle,\dots,$ and $\langle c-1,b+c-1,2b+c-1,3b+c-1,\dots,(a/c-1)b+c-1,c-1\rangle$.*

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