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Credibility decay model in temporal evidence combination ☆

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ABSTRACT

Data fusion in time domain is sequential and dynamic. Methods to deal with evidence conflict in spatial domain may not suitable in temporal domain. It is significant to determine the dynamic credibility of evidence in time domain. The Markovian requirement of time domain fusion is analyzed based on Dempster's combination rule and evidence discount theory. And the credibility decay model is presented to get the dynamic evidence credibility. Then the evidence is discounted by dynamic discount factor. It's illustrated that such model can satisfied the requirement of data fusion in time domain. Proper and solid decision can be made by this approach.

1. Introduction

The Dempster–Shafer theory of evidence, also called belief function theory, is an important method to deal with uncertainty in information systems. Since it was firstly presented by Dempster [1], and was later extended and refined by Shafer [2], the Dempster–Shafer theory, or the D–S theory for short, has generated considerable interest. Its application has extended to many areas such as expert systems, diagnosis and reasoning, pattern classification, information fusion, and data mining.

However, illogical results may be obtained by classical Dempster's combination rule when collected evidence highly conflicts each other. Many methods have been proposed to solve this problem, one of which is to modify the evidence. Evidence discounting and evidence averaging are typical methodologies to modify the evidence. The reliability of a source of information is classically taken into account by the discounting operation, which transforms a belief function into a weaker, less informative one. This operation is usually important in uncertain information management.

Most of previous research about evidence theory is carried out in spatial domain, where all the evidence bodies are combined simultaneously. This can be seen as a static fusion. However, influenced by noisy and the reliability of sensors, readings of sensors in one time period are not reliable enough for information fusion. So information from multiple periods is necessary for better fusion result. Thus temporal information fusion, also called dynamic fusion, is crucial to many applications. In dynamic fusion, belief functions are collected sequentially one by one. It is meaningless to combine all the belief functions after they were collected. Temporal fusion is sequential and dynamic, represented by the inheritance and update in fusion result.

In spatial evidence combination, belief functions are discounted or averaged by the credibility degrees or discounting factors obtained by analyzing the relationship between all belief functions. However, this method cannot make sense in temporal combination, where belief functions are not collected simultaneously. So how to determine the discount factor in temporal evidence fusion

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has received much attention. Although this problem has been addressed in [3], further mathematical implication and physical application are necessary for better imploration on the temporal evidence combination. Based on the time decay model proposed by Smets in [3], we give the definition of credibility decay model for evidence discounting in time domain, providing an alternative to analyzing and combining sequential evidences.

In the rest of this paper, background knowledge on evidence theory is briefly recalled firstly. Then the credibility decay model (CDM) is then presented after analysis on Markovian requirement of evidence combination rule in temporal domain based on the sequential characteristic of dynamic fusion. Finally, numerical simulation is carried out to illustrate the performance of proposed model.

2. Preliminaries

Dempster–Shafer theory of evidence was modeled based on a finite set of mutually exclusive elements, called the frame of discernment denoted by Ω [1]. The power set of Ω, denoted by 2^{Ω}, contains all possible unions of the sets in Ω including Ω itself. Singleton sets in a frame of discernment Ω will be called atomic sets because they do not contain nonempty subsets. The following definition is central in the Dempster–Shafer theory.

Definition 1. Let $\Omega = \{A_1, A_2, \cdots, A_n\}$ be the frame of discernment. A basic probability assignment (BPA) is a function m: $2^{\Omega} \to [0,1]$, satisfying the two following conditions:

$$m(\emptyset) = 0 \tag{1}$$

$$\sum_{A \subseteq \Omega} m(A) = 1 \tag{2}$$

where $\emptyset$ denotes empty set, and A is any subset of Ω. Such a function is also called a basic belief assignment by Smets [4], and a belief structure (BS) by Yager [5]. The terminology of belief function will be adopted in this paper. For each subset $A \subseteq \Omega$, the value taken by the BPA at A is called the basic probability assigned to A, or the BPA of A for short, denoted by $m(A)$.

Definition 2. A subset A of Ω is called the focal element of a belief function m if $m(A) > 0$.

Definition 3. Given a belief function m on Ω, the belief function and plausibility function which are in one-to-one correspondence with m can be defined respectively as:

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{3}$$

$$Pl(A) = \sum_{B \cap A \neq \emptyset} m(B) = 1 - \sum_{B \cap A = \emptyset} m(B) \tag{4}$$

Definition 4. (See [4].) The pignistic transformation maps a belief function m to so called pignistic probability function. The pignistic transformation of a belief function m on $\Omega = \{A_1, A_2, \cdots, A_n\}$ is given by

$$BetP(A) = \sum_{B \subseteq \Omega} \frac{|A \cap B|}{|B|} \frac{m(B)}{1 - m(\emptyset)}, \quad \forall A \subseteq \Omega \tag{5}$$

where $|A|$ is the cardinality of set A.

In a particular case where $m(\emptyset) = 0$ and $A \in \Omega$, i.e., A is a singleton of Ω, we have

$$BetP(A) = \sum_{A \in B} \frac{m(B)}{|B|}, \quad A = A_1, \ldots, A_n, \ B \subseteq \Omega \tag{6}$$

Definition 5. Given two belief functions m_1 and m_2 on Ω, the belief function that results from the application of Dempster's combination rule, denoted as $m_1 \oplus m_2$, or m_{12} for short, is given by:

$$m_1 \oplus m_2(A) = \begin{cases} \frac{\sum_{B \cap C = A} m_1(B) m_2(C)}{1 - \sum_{B \cap C = \emptyset} m_1(B) m_2(C)}, & A \subseteq \Omega, \ A \neq \emptyset \\ 0, & A = \emptyset \end{cases} \tag{7}$$

Definition 6. Let m be the BPA on the discriminant frame Ω. It is produced by a sensor S, with a reliability of α, $\alpha \in [0,1]$. Then m can be discounted as [2]:

$$m^{\alpha}(A) = \begin{cases} \alpha m(A), & A \neq \Omega \\ 1 - \alpha + \alpha m(A), & A = \Omega \end{cases} \tag{8}$$

Definition 7. The vacuous belief function on Ω is a categorical belief function focused on Ω, i.e. $m(\Omega) = 1$. It is denoted VBF.

VBF means full ignorance on Ω. As for Dempster combination rule, we can easily get: $m \oplus VBF = m$ or $f(m, VBF) = m$.

Eq. (8) also indicates that, $m^{\alpha}(A) \to VBF$ for $\alpha \to 0$, and $m^0 = VBF$. This means that the information provided by an unreliable sensor is total ignorance.

Lemma 1. $VBF^{\alpha} = VBF$.

Proof. Let $A = \Omega$ and $m(A) = 1$, given Eq. (8) we can get: $m^{\alpha}(A) = 1 - \alpha + \alpha = 1 = m(A)$. $\square$

Lemma 2. *Let m be BPA on* Ω, $\alpha_1, \alpha_2 \in [0,1]$, *then* $(m^{\alpha_1})^{\alpha_2} = m^{\alpha_1 \alpha_2}$.

Proof. Given Eq. (8), we have $m^{\alpha_1}(A) = \begin{cases} \alpha_1 m(A), & A \neq \Omega, \\ 1 - \alpha_1 + \alpha_1 m(A), & A = \Omega. \end{cases}$

For $A \neq \Omega$:

$$\begin{aligned} (m^{\alpha_1})^{\alpha_2}(A) &= \alpha_2 \cdot (\alpha_1 m(A)) \\ &= \alpha_1 \alpha_2 m(A) = m^{\alpha_1 \alpha_2}(A). \end{aligned}$$

For $A = \Omega$:

$$\begin{aligned} (m^{\alpha_1})^{\alpha_2}(A) &= \alpha_2 \cdot (1 - \alpha_1 + \alpha_1 m(A)) + 1 - \alpha_2 \\ &= 1 - \alpha_1 \alpha_2 + \alpha_1 \alpha_2 m(A) = m^{\alpha_1 \alpha_2}(A). \qquad \square \end{aligned}$$