Available online at www.sciencedirect.com
SCIENCE DIRECT®
Information Processing Letters 95 (2005) 363–369
Information Processing Letters
www.elsevier.com/locate/ipl

# Completeness of hyper-resolution via the semantics of disjunctive logic programs

Linh Anh Nguyen [a,*,1], Rajeev Goré [b]

[a] *Institute of Informatics, University of Warsaw, ul. Banacha 2, 02-097 Warsaw, Poland*
[b] *RSISE and NICTA, The Australian National University, Canberra ACT 0200, Australia*

Received 28 November 2004; received in revised form 2 February 2005; accepted 2 February 2005

Available online 23 May 2005

Communicated by J. Chomicki

**Abstract**

We present a proof of completeness of hyper-resolution based on the fixpoint semantics of disjunctive logic programs. This shows that hyper-resolution can be studied from the point of view of logic programming.


*Keywords:* Fixpoint semantics; Automatic theorem proving

## 1. Introduction

Resolution was introduced by Robinson in his landmark paper [24] in 1965 as a mechanizable method for detecting the unsatisfiability of a given set of formulae of classical first-order logic. It revolutionized the field of automated reasoning, and since then, many refinements of resolution have been proposed by researchers in the field in order to cut down the search space and increase efficiency. One of the most important refinements of resolution is hyper-resolution, which was also introduced by Robinson [23] in the same year 1965. Hyper-resolution constructs a resolvent of a number of clauses at each step. Thus it contracts a sequence of bare resolution steps into a single inference step and eliminates interactions among intermediary resolvents, and interactions between them and other clauses.

Resolution and hyper-resolution have been well studied. There are a number of well-known proofs of completeness of resolution and hyper-resolution. The classical proofs of completeness of hyper-resolution are often based on proofs of completeness of resolution. In Leitsch's book on resolution [12], complete-

* Corresponding author.
*E-mail addresses:* nguyen@mimuw.edu.pl (L.A. Nguyen), rajeev.gore@anu.edu.au (R. Goré).

[1] Partially supported by a visiting fellowship from NICTA. National ICT Australia (NICTA) is funded through the Australian Government's *Backing Australia's Ability* initiative, in part through the Australian Research Council.

0020-0190/$ – see front matter 
doi:10.1016/j.ipl.2005.02.003

ness results are proved using the semantic tree technique [11]. Some other methods for proving completeness of resolution and hyper-resolution are Bachmair and Ganzinger's forcing technique [3], de Nivelle's resolution game technique [7], Boyer's excess literal technique (see [6]), and a proof-theoretic method by Goubault-Larrecq [9].

There is a close relationship between the theory of logic programming and resolution. A refinement of resolution for the Horn fragment, called SLD-resolution in [1], was first described by Kowalski [10] for logic programming. It is a top-down procedure for answering queries in definite logic programs. On the other hand, a bottom-up method for answering queries is based on fixpoint semantics of logic programs and was first introduced by van Emden and Kowalski [25] using the direct consequence operator $T_P$. This operator is monotonic, continuous, and has the least fixpoint $T_P \uparrow \omega = \bigcup_{n=0}^{\omega} T_P \uparrow n$, which forms the least Herbrand model of the given logic program $P$.

In [16], Minker and Rajasekar extended the fixpoint semantics to disjunctive logic programs. Their direct consequence operator, denoted by $T_P^I$, iterates over model-states, which are sets of disjunctions of ground atoms. This operator is also monotonic, continuous, and has a least fixpoint which is a least model-state characterizing the given program $P$.

Fixpoint semantics of logic programs are closely related to hyper-resolution. More precisely, the intuition behind hyper-resolution is the same as that behind the fixpoint semantics of disjunctive logic programs, as has been observed by researchers in logic programming. There are, however, the following differences: a disjunctive logic program is a set of *non-negative* clauses, and the direct consequence operator (as defined in [16,14]) *simultaneously* applies inference steps for all *ground* instances of clauses.

Despite the fact that a number of resolution systems have been implemented in Prolog, the relationship between the theory of logic programming and the theory of resolution has been, in our opinion, mostly one-directional: logic programming has been studied from the point of view of resolution. Our thesis is that hyper-resolution can be studied from the point of view of logic programming. In this paper, we give a proof of completeness of hyper-resolution based on the fixpoint semantics of disjunctive logic programs.

## 2. Preliminaries

First-order logic is considered in this work and we assume that the reader is familiar with it. We assume we are given a finite set of first-order formulae, which we wish to test for satisfiability. We now give the most important definitions for our work.

### 2.1. Herbrand models

Let $L$ be the underlying first order language for the considered set of formulae. Normally, we assume that $L$ is defined by the constants, function symbols and predicate symbols appearing in the considered set of formulae.

The *Herbrand universe* $U_L$ for $L$ is the set of all ground terms, which can be formed out of the constants and function symbols appearing in $L$. If $L$ has no constants, we add some constant, say $a$, to form ground terms.

An *Herbrand interpretation* for $L$ is an interpretation for $L$ such that:

- The domain of the interpretation is the Herbrand universe $U_L$;
- Constants in $L$ are assigned themselves in $U_L$;
- If $f$ is an $n$-ary function symbol in $L$, then the mapping from $(U_L)^n$ to $U_L$ defined by $(t_1, \ldots, t_n) \to f(t_1, \ldots, t_n)$ is assigned to $f$.

Since the assignment to constants and function symbols is fixed in Herbrand interpretations, we can identify an Herbrand interpretation with the set of all ground atoms which are true with respect to that interpretation.

Let $S$ be a set of closed formulae of $L$. An *Herbrand model* of $S$ is an Herbrand interpretation for $L$ which is a model for $S$.

### 2.2. Unification

A *substitution* is a finite set $\theta = \{x_1 := t_1, \ldots, x_n := t_n\}$, where $x_1, \ldots, x_n$ are different variables, $t_1, \ldots, t_n$ are terms, and $t_i \neq x_i$ for all $1 \leqslant i \leqslant n$. By $\varepsilon$ we denote the *empty substitution*.

An *expression* is either a term or a formula without quantifiers, and a *simple expression* is either a term or an atom.