Available online at www.sciencedirect.com
SCIENCE DIRECT®

Information Processing Letters 94 (2005) 119–123

Information Processing Letters

www.elsevier.com/locate/ipl

An LP rounding algorithm for approximating uncapacitated facility location problem with penalties ☆

Guang Xu *, Jinhui Xu

Department of Computer Science and Engineering, State University of New York at Buffalo, Buffalo, NY 14260, USA

Received 9 August 2004; received in revised form 28 December 2004

Communicated by F.Y.L. Chin

Abstract

In this paper, we consider an interesting variant of the facility location problem called *uncapacitated facility location problem with penalties* (UFLWP, for short) in which each client can be either assigned to some opened facility or rejected by paying a penalty. Existing approaches [M. Charikar, S. Khuller, D. Mount, G. Narasimhan, Algorithms for facility location problems with outliers, in: Proc. Symposium on Discrete Algorithms, 2001, p. 642] and [K. Jain, M. Mahdian, E. Markakis, A. Saberi, V. Vazirani, Greedy facility location algorithms analyzed using dual fitting with factor-revealing LP, J. ACM 50 (2003) 795] for this variant of facility location problem are all based on primal-dual method. In this paper, we present an efficient linear programming (LP) rounding based approach to show that LP rounding techniques are equally capable of solving this variant of facility location problem. Our algorithm uses a two-phase filtering technique (generalized from Lin and Vitter's [ε-approximation with minimum packing constraint violation, in: Proc. 24th Annual ACM Symp. on Theory of Computing, 1992, p. 771]) to identify those to-be-rejected clients and open facilities for the remaining ones. Our approach achieves an approximation ratio of $2 + 2/e$ (≈ 2.736) which is worse than the best approximation ratio of 2 achieved by the much more sophisticated dual fitting technique [K. Jain, M. Mahdian, E. Markakis, A. Saberi, V. Vazirani, Greedy facility location algorithms analyzed using dual fitting with factor-revealing LP, J. ACM 50 (2003) 795], but better than the approximation ratio of 3 achieved by the primal-dual method [M. Charikar, S. Khuller, D. Mount, G. Narasimhan, Algorithms for facility location problems with outliers, in: Proc. Symposium on Discrete Algorithms, 2001, p. 642]. Our algorithm is simple, natural, and can be easily integrated into existing LP rounding based algorithms for facility location problem to deal with outliers.
© 2005 Elsevier B.V. All rights reserved.

Keywords: Algorithms; Approximation algorithms; Facility location problem; Outliers

☆ This research was supported in part by an IBM faculty partnership award, and an IRCAF award from SUNY Buffalo.
* Corresponding author.
E-mail addresses: guangxu@cse.buffalo.edu (G. Xu), jinhui@cse.buffalo.edu (J. Xu).

0020-0190/$ – see front matter © 2005 Elsevier B.V. All rights reserved.
doi:10.1016/j.ipl.2005.01.005

1. Introduction

Facility location problem is a fundamental problem in operations research and theoretical computer science. A number of efficient approaches have been proposed in recent years, which can be roughly classified into three categories [11], LP rounding techniques [2,12], local search heuristics [1,3], and primal-dual method [7]. In some sense, techniques from different categories complement each other, and could be combined to achieve improved approximation algorithms [3,5].

In this paper, we study an interesting variant of the facility location problem called *uncapacitated facility location problem with penalties* (UFLWP, for short). Unlike the uncapacitated facility location problem, the UFLWP problem does not require that every client should receive its service from some opened facility. Instead, the demands from a client can be rejected by paying a penalty. The objective of the UFLWP problem is to determine a set of clients whose demands should be rejected and to assign the demands of the remaining clients to some opened facilities so as to minimize the total cost, including the penalties caused by rejecting demands from clients, the facility cost, and the assignment cost. The UFLWP problem provides an effective way to model those facility location problems in which a small portion of distant clients (viewed as outliers) can increase the overall cost drastically and finds its applications in majority of the commercial applications of the uncapacitated facility location problem [4].

So far, two constant approximation algorithms were proposed for solving this problem [4,8]. In [4], Charikar et al. obtained a 3-approximation algorithm by using the primal-dual method. Later on, Jain et al. used an elegant dual fitting technique to achieve a 2-approximation combinatorial algorithm [8]. Both algorithms fall in the category of primal-dual method. Thus it would be very interesting to know whether techniques from other categories are also capable of solving this variant of facility location problem since such techniques, if they exist, might be combined with the primal-dual methods to further improve the approximation ratio of the UFLWP problem. In this paper, we give an affirmative answer to this question by presenting an LP rounding based $(2 + 2/\mathrm{e})$-approximation algorithm for the UFLWP problem, where e is the natural logarithmic base. Our algorithm uses a two-phase filtering technique generalized from the one introduced by Lin and Vitter [10]. Our algorithm identify the set of to-be-rejected clients in the first phase of the filtering and rounding procedure. In the second phase, the remaining clients are assigned to some opened facilities. We notice that the filtering technique has been shown to be a very effective method for solving facility location problems and some other NP-hard problems [12,2,9,6]. The performance ratio of our algorithm is better than that of [4], but worse than the performance ratio of [8]. Our algorithm is simple, natural, and can be easily integrated into existing LP rounding based algorithms for other variants of the facility location problem to extend their ability to deal with outliers. The performance ratio of our algorithm could be further improved if a better LP rounding based algorithm for the uncapacitated facility location problem is discovered in the future.

2. Algorithm for uncapacitated facility location problem with penalties

2.1. Problem description

The uncapacitated facility location problem with penalties can be stated formally as follows. We are given a bipartite graph G with bipartition $(\mathcal{F}, \mathcal{C})$, where $\mathcal{F}$ is the set of locations where facilities can be opened and $\mathcal{C}$ is the set of locations (called clients) where services are demanded. The demands of a client could be either satisfied by assigning the client to some opened facility or rejected. Each location $F_i \in \mathcal{F}$ is associated with a nonnegative opening cost f_i. Each client $C_j \in \mathcal{C}$ is associated with two positive numbers d_j and r_j. d_j specifies the amount of demands required by client C_j. The assigning cost for connecting C_j to an opened location F_i is $d_j c_{ij}$, where c_{ij} is the distance between F_i and C_j. In this paper, we consider the metric version of this problem, i.e., $\forall F_i$, $F_{i'} \in \mathcal{F}$, $C_j, C_{j'} \in \mathcal{C}$, $c_{ij} \leqslant c_{i'j} + c_{ij'} + c_{i'j'}$. r_j is the cost (or penalty) if the demands from client C_j are rejected. The goal of this problem is to find

(1) a subset $R \subseteq \mathcal{C}$ of clients whose demands should be rejected,

Download English Version:

https://daneshyari.com/en/article/10331987

Download Persian Version:

https://daneshyari.com/article/10331987

Daneshyari.com