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Optimal price, warranty length and production rate for free replacement policy in the static demand market☆

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Abstract

In the present paper, a decision model is developed for producers in the static demand market to determine the optimal price, warranty length and production rate of a product to maximize profit based on the pre-determined life cycle. The free renewal warranty policy is considered under which failed products are renewed before the end of warranty length at no cost to consumers. The expected number of renewals based on warranty length is derived for Weibull life distributed products. The objective function includes both demand and cost functions, where production cost, warranty cost and inventory cost are involved. A solution approach using the maximum principle is described, and is applied to two specific cases of markets. The first case of market considers positive discount rate, and the second case of market considers zero discount rate. The economic sensitivity analysis is conducted to evaluate the effect of model parameters on the optimal solution. Some conclusions are drawn based on the sensitivity analysis.

Keywords: Free renewal; Price; Production rate; Warranty length; Weibull distribution

1. Introduction

For the same class of products in the market, lower price usually tends to enhance sales volume, but leads to a decrease in the unit profit. Pricing must be based on cost to create profit; meanwhile, the price of a product should not be too high to dissuade consumers from purchasing. Therefore, producers have to treat product pricing as a competitive tool in their marketing strategy [1].

In addition to price, consumers may predict the quality of a product based on its warranty, which is considered as the assurance that the producer provides after evaluating the strength of products [2]. For promotion, warranty can be considered as a marketing tool to differentiate from competitors [3,4], since a satisfactory warranty policy will certainly enhance consumers' purchase willingness. If the producer promises to renew or repair products when failures occur, the commitment length of warranty and the reliability of the product, which is related to its lifetime distribution, play a key role on determining the total cost of product.

A producer must strive to obtain maximum profit in the long run which depends not only on the marketing

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policy (i.e. price and warranty length) but also production rate. Production rate is defined as the number of items produced over a given time period and should be used as a decision variable for profit maximization [5–7]. The machine production rate may be easily adjusted to satisfy the market demand. That is, different levels of demand can be satisfied with appropriate production rates. This implies that production rate is very important for customer satisfaction as well as from the producer's point of view.

In the present paper, we deal with the problem of determining the optimal price, warranty length and production rate for a product with a Weibull lifetime distribution in the static demand market such that the present value of product profit may be maximized. The Weibull distribution is used as a very popular parametric family of failure distributions in reliability studies [8,9] and is able to describe the increasing and decreasing failure rates in addition to the constant failure. Meanwhile, use of discounted cash flows presents the advantage of being within a framework that is common in studies related to long-term financial decisions [10,11]. The static demand market, which has been investigated by Teng and Thompson [12], characterizes the market that word of mouth is not important and represents the case that demand function exhibits no learning or saturation.

In the next section, we consider the free renewal warranty policy under which the failed products are renewed before the end of warranty length at no cost to consumers. Moreover, the demand function of the static market, and the cost function will be discussed. The decision model with price, warranty length and production rate will then be developed, and the model components, including the expected renewal cost, and the solution approach will be described in detail. The economic sensitivity analysis will be presented to evaluate the effect of model parameters and market characteristics on the optimal solution of the decision model, and finally some conclusions will be drawn based on the analysis.

2. Development of the decision model

The decision model for determining the optimal price, warranty length and production rate includes two key components: the demand function and the cost function. Both functions are briefly reviewed and discussed in this section.

2.1. The demand function

The static market is for non-durable products where diffusion effect is not important, e.g. inexpensive or established products. In the present paper, the static market is considered where demand exhibits no 'learning' or saturation, and is applied to two types of market static. The first type of market static considers the case of positive discount rate, which means that one dollar today has more worth than one dollar in the future. The second type of market static considers the case of zero discount rate, which means that one dollar today has the same value as one dollar in the future.

Glickman and Berger [13] presented a demand function of market static in which the demand decreases exponentially with respect to price and increases exponentially with warranty length. Denote p and w as the price and warranty length of product, respectively. The demand function, $f(p, w)$, in Glickman and Berger's model [13] is a displaced log-linear function with an exponential form as follows:

$$f(p, w) = k_1 p^{-a}(w + k_2)^b, \tag{1}$$

where k_1 is a constant of amplitude factor, k_2 is a constant of time displacement that allows for the possibility of non-zero demand when w is zero, a is the price elasticity, and b is the displaced warranty length elasticity. Both k_1 and k_2 are greater than zero, the value of a is greater than one, and the value of b is between zero and one. It can be seen that any increase in a downward shifts in demand, which is consistent with the behavior of marginal profit curve. For practical mathematical manipulation, Eq. (1) may be solved with ease by applying logarithm operations. Applications of this demand function could be found in the literature. For example, Mitra and Jayprakash [14] presented a multi-objective model for warranty estimation based on this demand function; Blischke and Murthy [15] used this demand function in product warranty management; Mitra and Jayprakash [16] also applied this demand function to develop a market-share model.

2.2. The cost function

The cost function involves production and warranty cost and inventory cost. These cost components are described in this section.

2.2.1. Production and warranty cost

To determine the reasonable price, warranty length and production rate for a product, producer must estimate the corresponding cost, including production cost and the cost of providing warranty. Let the unit cost function $C(w(t), q(t))$ be a function of warranty length at time t, $w(t)$, and production rate at time t, $q(t)$.