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JOCS-227; No. of Pages 12
ARTICLE IN PRESS

Journal of Computational Science xxx (2013) xxx–xxx

Contents lists available at ScienceDirect

Journal of Computational Science

journal homepage: www.elsevier.com/locate/jocs

Modal analysis reduction of multi-body systems with generic damping

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A R T I C L E I N F O

Article history:
Received 2 February 2013
Received in revised form 25 June 2013
Accepted 25 August 2013
Available online xxx

Keywords:
Lumped-parameter systems
Multi-body systems
Non-proportional damping
Modal analysis
Model reduction

A B S T R A C T

Modal analysis of multi-body systems is broadly used to study the behavior and controller design of dynamic systems. In both cases, model reduction that does not degrade accuracy is necessary for the efficient use of these models. Previous work by the author addressed the reduction of modal representations by eliminating entire modes or individual modal elements (inertial, compliant, resistive). In that work, the bond graph formulation was used to model the system and the modal decomposition was limited to systems with proportional damping. The objective of the current work is to develop a new methodology such that model reduction can be implemented to modal analysis of multi-body systems with non-proportional damping that were not modeled using bond graphs. This extension also makes the methodology applicable to realistic systems where the importance of modal coupling terms is quantified and potentially eliminated. The new methodology is demonstrated through an illustrative example.

1. Introduction

Current modeling techniques and simulation tools provide engineers with a variety of options when it comes to modeling of new or existing systems. These tools and techniques are powerful and extensively used in everyday engineering, nevertheless further improvements on modeling decisions and model complexity issues would make them more efficient. Specifically, a main disadvantage is that modeling techniques and simulation tools require sophisticated users who are often not domain experts and thus lack the ability to effectively utilize the available tools to uncover the important design trade-offs. Another drawback is that models are often large and complicated with many parameters, making the physical interpretation of the model outputs, even by domain experts, difficult. This is particularly true when "unnecessary" features are included in the model.

A variety of algorithms have been developed and implemented to help automate the production of proper models of dynamic systems. Wilson and Stein [1] developed the model order deduction algorithm (MODA) that deduces the required system model complexity from subsystem models of variable complexity using a frequency-based metric. They also defined proper models as the models with physically meaningful states and parameters that are of necessary but sufficient complexity to meet the engineering and accuracy objectives. Additional work on deduction algorithms for generating proper models in an automated fashion has extended the functionality of MODA [2–4]. These algorithms have also been implemented and demonstrated in an automated modeling environment [5].

In an attempt to overcome the limitations of the frequency-based metrics, Louca et al. [6] introduced a new model reduction technique that also generates proper models. This approach uses an energy-based metric (element activity) that in general, can be applied to nonlinear systems [7], and considers the importance of all energetic elements (generalized inductance, capacitance and resistance). The contribution of each energy element in the model is ranked according to the activity metric under specific excitation. Elements with small contribution are eliminated in order to produce a reduced model. The activity metric was also used as a basis for even further reduction, through partitioning a model into smaller and decoupled submodels [8].

Beyond the physical-based model reduction, modal decomposition can also be used to model and analyze continuous and discrete systems [9–12]. One of the advantages of modal decomposition is the ability to directly adjust (i.e., reduce) model complexity since all modes are orthogonal to each other. The reduction of such modal decomposition models is mostly based on frequency, and the user defined frequency range of interest (FROI) determines the frequencies that are important for a specific scenario. In this case, modes with frequencies within the FROI are retained in the reduced model and modes outside this range are eliminated. As expected, mode truncation introduces error in the predictions that can be measured and adjusted based on the accuracy requirements [13,14].

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http://dx.doi.org/10.1016/j.jocs.2013.08.008

The element activity metric provides more flexibility than frequency-based metrics, which address the issue of model complexity by only the frequency content of the model. In contrast, the activity metric considers the energy flow in the system, and therefore, the importance of all energy elements in the model can be described. Previous work by the author addressed the development and reduction of modal representations using the bond graph formulation and using the activity metric [15,16]. This work introduced a methodology that reduces the model complexity by eliminating entire modes or partial modes through modal elements (inertial, compliant, resistive). The identification and elimination of insignificant elements was performed with the use of the activity metric and model order reduction algorithm (MORA). This approach has advantages over frequency-based reduction techniques; however, it has a significant limitation in that it can only be applied to systems with proportional damping and thus is not able to be applied to realistic systems.

The objective of the current work is to develop a methodology that overcomes the limitations of the author's previous work, such that modal analysis and model reduction can be applied to a more general class of systems with non-proportional damping. In addition, the activity metric will be formulated for systems that are modeled with second order ordinary differential equations (ODE), rather than bond graphs and first order ODEs that were used in previous work. Second order ODEs are typically derived from Lagrange's equations or Newton's law. These two additions will make the activity metric a more appealing model reduction methodology that can be applied to realistic systems.

This paper is organized as follows: first, background about the energy-based metric and modal decomposition are provided. Next, the equation formulation and modal decomposition of multi-body systems with non-proportional damping is presented. Then, the activity analysis of all modal elements is introduced, along with the closed-form expressions of the steady state activities. An illustrative example of a linear quarter car model is also presented, in order to demonstrate the development of its modal decomposition and the evaluation of the coupling terms' importance using the activity metric. Finally, in the last section, discussion and conclusions are given.

2. Background

2.1. Energy-based model reduction

The original work on the energy-based metric for model reduction is briefly described here since it is the foundation of the contributions in this paper. The main idea behind this model reduction technique is to evaluate the "element activity" of the individual energy elements of a complex system model under a stereotypic set of inputs and initial conditions. The activity of each energy element establishes an importance hierarchy for all elements. Those below a user-defined threshold of acceptable level of activity are eliminated from the model. A reduced model is then generated and a new set of governing differential equations is derived. More details, extensions, and applications of this approach are given in previous publications [7,17–19].

The activity metric has been previously formulated for systems with nonlinearities in both the element constitutive laws and kinematics. In this work, the activity metric is applied to linear systems for which analytical expressions for the activity can be derived, and therefore, avoid the use of numerical time integration that could be cumbersome. The analysis is further simplified if, in addition to the linearity assumption, the system is assumed to have a single sinusoidal excitation, and only the periodic steady state response is examined. These assumptions are motivated from Fourier analysis where an arbitrary function can be decomposed into a series of harmonics. Using this decomposition, the activity analysis can be performed as a function of frequency in order to study the frequency dependency of the energy elements in a dynamic system.

2.1.1. Element activity for linear systems

The starting point of a model reduction process is a system model, which typically includes complexity that has minimal contribution to the accuracy of the model's dynamic response. Thus, the goal of a reduction algorithm is to identify this "unnecessary" complexity in order to generate a reduced model that is easier to analyze yet accurate. One approach to accomplish this is the activity metric that was previously defined by the author [7]. The activity metric is outlined below as it was originally developed for models of multi-energy systems that are represented by first order ODEs.

Models of dynamic systems consist of physical energy elements that can store (inertia and stiffness) or dissipate (resistance) energy. In addition, these can be considered as generalized elements in order to have the ability to model multi-energy systems, i.e., translational mechanical, rotational mechanical, electrical and hydraulic. A quantity that can be defined for all energy elements in a model and it is independent of its energy domain, is power. However, power is time dependent and thus not a suitable modeling metric.

A measure of the power response of a dynamic system, which has physical meaning and a simple definition, is used to develop the modeling metric, element activity (or simply "activity"). Element activity, A, is defined for each energy element as:

$$A = \int_0^{\tau} \left|\mathbb{P}(t)\right| \cdot dt \qquad (1)$$

where $\mathbb{P}(t)$ is the element power and τ is the time over which the model has to predict the system behavior. The activity has units of energy, representing the amount of energy that flows in and out of the element over the given time τ. The energy that flows in and out of an element is a measure of how active this element is (how much energy passes through it), and consequently the quantity in Eq. (1) is termed activity.

Element power is the product of generalized effort and flow, which for the linear mechanical domain is the product of force and velocity. Given this definition the activity can be rewritten as:

$$A = \int_0^{\tau} \left|\mathbb{P}(t)\right| \cdot dt = \int_0^{\tau} \left|F(t) \cdot v(t)\right| \cdot dt \qquad (2)$$

The multi-body system is assumed to have k_m mass, k_k stiffness and k_b damping elements, thus, $k_e = k_m + k_k + k_b$ energy elements. Element power is calculated by using either the effort or flow and the constitutive law, and therefore, only k_e outputs are required for calculating the activity of all elements in the system. The outputs are selected to be velocity, force, and velocity for mass, stiffness, and damping elements, respectively. The duals of these variables can also be used for calculating element power without loss of generality. Also, the parameters m, k and b are known constants representing the linear constitutive law coefficients of mass, stiffness and damping, respectively. With the above definitions, the power of each energy element is calculated as:

$$\begin{aligned} &\text{Mass}: \quad \mathbb{P}_m = F_m \cdot v_m = (m \cdot \dot{v}_m) \cdot v_m \\ &\text{Stiffness}: \quad \mathbb{P}_k = F_k \cdot v_k = F_k \cdot \left(\frac{\dot{F}_k}{k}\right) \\ &\text{Damping}: \quad \mathbb{P}_b = F_b \cdot v_b = (b \cdot v_b) \cdot v_b \end{aligned} \qquad (3)$$