Journal of Computer and System Sciences 82 (2016) 767–781

Contents lists available at ScienceDirect

Journal of Computer and System Sciences

www.elsevier.com/locate/jcss

Edge-fault-tolerant pancyclicity and bipancyclicity of Cartesian product graphs with faulty edges ☆

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ARTICLE INFO

Article history:
Received 23 October 2014
Received in revised form 16 November 2015
Accepted 4 January 2016
Available online 10 March 2016

Keywords:
Cartesian product graphs
Edge-bipancyclic
Edge-pancyclic
Fault-tolerant embeddings
Interconnection networks

ABSTRACT

Let $r \geq 4$ be an even integer. Graph G is *r-bipancyclic* if it contains a cycle of every even length from r to $2\left\lfloor \frac{n(G)}{2} \right\rfloor$, where $n(G)$ is the number of vertices in G. A graph G is *r-pancyclic* if it contains a cycle of every length from r to $n(G)$, where $r \geq 3$. A graph is *k-edge-fault Hamiltonian* if, after deleting arbitrary k edges from the graph, the resulting graph remains Hamiltonian. The terms *k-edge-fault r-bipancyclic* and *k-edge-fault r-pancyclic* can be defined similarly. Given two graphs G and H, where $n(G), n(H) \geq 9$, let $k_1, k_2 \geq 5$ be the minimum degrees of G and H, respectively. This study determined the edge-fault r-bipancyclic and edge-fault r-pancyclic of Cartesian product graph $G \times H$ with some conditions. These results were then used to evaluate the edge-fault pancyclicity (bipancyclicity) of $NQ_{m_r,\ldots,m_1}$ and $GQ_{m_r,\ldots,m_1}$.

1. Introduction

A *multicomputer system* comprises multiple processors (nodes) that utilize its underlying topology called an *interconnection network* (*network* for brevity) for exchanging messages. Every processor contains one or more processing elements, as well as a communication modules and a local memory. An interconnection network is usually represented by a simple, connected, and undirected graph $G = (V, E)$ in which the processors are represented by the *vertex* (*node*) *set* V and the communication links are represented by the *edge set* E.

A *Cartesian product graph* (*product graph* for brevity) can be constructed by applying the graph Cartesian product operation "$\times$" to factor graphs. Applying the Cartesian product to combine two graphs with established properties enables the construction of a new graph with the properties of both graphs. For example, many distributed memory multiprocessors, such as the Cray T3D [19] and the Cray T3E have been built with k-ary n-cubes (a subclass of nearest neighbor mesh hypercubes) forming the underlying topology [26]. Numerous topological properties of Cartesian product graphs have been investigated in the literature, including embedding, shortest-path routing, diameter, and connectivity [12–15,21,27,28,32].

Rings (*cycles*) form the most basic class of network topologies for designing parallel and distributed algorithms with low communication costs. Numerous efficient algorithms were proposed based on rings to solve various problems [1,22]. Recently, [3,18] have determined results for cycle embedding. Graph G is *k-pancyclic* if it contains a cycle of every length from k to $n(G)$ for any $k \geq 3$, or *k-bipancyclic* if it contains a cycle of every even length from k to $n(G)$ for any even

☆ This work was supported in part by the Ministry of Science and Technology, Taiwan under grant NSC 100-2628-E-006-027-MY3, and by (received funding from) the Headquarters of University Advancement at the National Cheng Kung University, which is sponsored by the Ministry of Education, Taiwan, ROC.

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http://dx.doi.org/10.1016/j.jcss.2016.01.003

Table 1
Summary of related works.

Network	# of faulty vertices/edges	Property	Ref.
An n-dimensional hypercube for $n \geq 3$, where every vertex is incident with at least two non-faulty edges	$2n-5$ edges	bipancyclic	[29]
An n-dimensional crossed cube for $n \geq 3$	$n-2$ elements	pancyclic	[33]
An n-dimensional twisted cube for any odd integer $n \geq 3$	$n-2$ elements	pancyclic	[8,34]
An n-dimensional Möbius cube for $n \geq 2$	$n-2$ elements	pancyclic	[16]
An n-dimensional augmented cube for $n \geq 4$	$2n-3$ elements	pancyclic	[30]
An n-dimensional star graph for $n \geq 3$	$n-3$ edges	bipancyclic	[23]
An n-dimensional bubble-sort graph for $n \geq 4$	$n-3$ edges	bipancyclic	[20]
An n-dimensional alternating group graph for $n \geq 4$	$n-2$ vertices	pancyclic	[9]
An n-dimensional locally twisted cube for $n \geq 3$	$n-2$ elements	pancyclic	[7]
A k-ary n-cube for $n \geq 3$ and any even $k \geq 4$, where every vertex is incident with at least two non-faulty edges	$4n-5$ edges	bipancyclic	[35]
A k-ary n-cube for $n \geq 3$ and any odd $k \geq 3$, where every vertex is incident with at least two non-faulty edges	$4n-5$ edges	k-pancyclic and bipancyclic	[25]

$k \geq 4$, where $n(G)$ is the *order* of G (i.e., the number of vertices in G). In addition, a 3-pancyclic graph is also called pancyclic and a 4-bipancyclic graph is also called bipancyclic. The bipancyclicity or the pancyclicity of a given network is among the most critical factors for determining whether a network's topology can emulate rings of various lengths [36,37]. Because link (edge) or vertex faults can occur when a network is activated, faulty networks must be considered. The pancyclicity and bipancyclicity of various faulty networks have received considerable attention in recent years [29,33,8,34,7,16,30,23,20,9,35,25]. Table 1 summaries several related works. Quite recently, Hsieh and Cheng [17] showed the edge-fault edge-bipancyclicity and edge-fault edge-pancyclicity of Cartesian product graphs $G \times H$. However, the topological property discussed in [17] is different from that studied in this paper. Furthermore, the number of tolerated faulty edges and the range of embedded fault-free cycle length in this paper are both larger than those considered in [17]. A *Hamilton decomposition* (also called a *Hamiltonian decomposition* [5]) of a Hamiltonian regular graph is a partition of its edge set into Hamiltonian cycles. Bermond [4] provided a conjecture that states: If G and H can be decomposed into Hamiltonian cycles, so does their Cartesian product. Note that Hamiltonian decomposability is stronger than edge-fault-Hamiltonian in the sense that an r-regular graph that is Hamiltonian decomposable is $(r-1)$-edge-fault-Hamiltonian. Alspach et al. [2] showed that every Q_n for $n > 2$ admits a Hamilton decomposition.

Given two graphs G and H, where $n(G) \geq 9$ and $n(H) \geq 9$, let $k_1 \geq 5$ and $k_2 \geq 5$ be the minimum degrees of G and H, respectively. This study determined the following two results: 1) The Cartesian product graph $G \times H$ is (k_1+k_2-2)-edge-fault r-bipancyclic and (k_1+k_2-2)-edge-fault Hamiltonian if G is (k_1-2)-edge-fault r_1-bipancyclic and (k_1-2)-edge-fault Hamiltonian and H is (k_2-2)-edge-fault r_2-bipancyclic and (k_2-2)-edge-fault Hamiltonian, where $\max\{r_1, r_2\} \leq \min\{n(G), n(H)\}$ and $r = \min\{r_1, r_2\} \geq 4$ for r_1 and r_2 are two even integers. 2) The Cartesian product graph $G \times H$ is (k_1+k_2-2)-edge-fault r-pancyclic if G is (k_1-2)-edge-fault r_1-pancyclic and H is (k_2-2)-edge-fault r_2-pancyclic, where $r = \min\{r_1, r_2\} \geq 3$ and $\max\{r_1, r_2\} \leq \min\{n(G), n(H)\}$. In the proposed approach, many small cycles in different subgraphs are first constructed, and then merged to form cycles with the desired length. When $k_1 + k_2 - 1$ faulty edges are all incident to a common vertex with the degree $k_1 + k_2$, a fault-free Hamiltonian cycle cannot be found. Hence, the obtained results are optimal for the number of faulty edges tolerated. The first result of this study is used to evaluate the edge-fault bipancyclicity of nearest neighbor mesh hypercubes, and the second result is used to evaluate the edge-fault pancyclicity of the nearest neighbor mesh hypercubes and the generalized hypercubes. According to a through review of relevant literature, this paper is the first to report these results.

The remainder of this paper is organized as follows. Section 2 provides definitions and notations. In Section 3, the edge-fault bipancyclicity of product graphs is evaluated, and in Section 4, the edge-fault pancyclicity of product graphs is evaluated. In Section 5, the results obtained from Section 3 and Section 4 are applied to evaluate edge-fault pancyclicity or bipancyclicity of generalized hypercubes and nearest neighbor mesh hypercubes. Section 6 concludes the paper with closing remarks.

2. Preliminaries

A *graph* $G = (V, E)$ comprises the *vertex set* V and the *edge set* E, where V is a finite set and E is a subset of $\{(u, v)|\ (u, v)$ is an unordered pair of $V\}$. The terms $V(G)$ and $E(G)$ are used to denote the vertex set and edge set of G, respectively. If (u, v) is an edge in G, then vertices u and v are *adjacent*. The edge (u, v) is *incident to* u and v, and u and v are the *endpoints* of (u, v). For the endpoints of an edge, one is a *neighbor* of the other. For a vertex v in G, the term $N_G(v)$ is used to denote the neighbors of v. The *degree* of vertex v, denoted by $d_G(v)$, is the number of edges incident to it. Let $\delta(G) = \min\{d_G(v)|\ v \in V(G)\}$. If $d_G(v) = \delta(G)$ for all $v \in V_G$, graph G is $\delta(G)$-*regular*.

A *path* $P[v_0, v_t] = \langle v_0, v_1, \ldots, v_t \rangle$ is a sequence of distinct vertices such that any two consecutive vertices are adjacent, where v_0 and v_t are the *endpoints* of $P[v_0, v_t]$. A path may contain a *subpath*, denoted by $\langle v_0, v_1, \ldots, v_i, P[v_i, v_j], v_j, v_{j+1}, \ldots, v_t \rangle$, where $P[v_i, v_j] = \langle v_i, v_{i+1}, \ldots, v_{j-1}, v_j \rangle$. A *cycle* $\langle v_0, v_1, \ldots, v_t, v_0 \rangle$ for $t \geq 2$ is a sequence of vertices such that