Available online at www.sciencedirect.com
ScienceDirect
Omega 35 (2007) 432–450
omega
The International Journal of Management Science
www.elsevier.com/locate/omega

Interactive fuzzy programming for decentralized two-level linear fractional programming (DTLLFP) problems

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Received 26 February 2004; accepted 31 August 2005
Available online 25 October 2005
Processed by W. Kubiak

Abstract

This paper presents two new interactive fuzzy programming approaches for a decentralized two-level linear fractional programming (DTLLFP) problem with a single decision maker (DM_0) at the upper level and multiple DMs ($DM_i,\ i=1,\ldots,k$) at the lower level. In the first approach, DM_0 specifies the minimal satisfactory level for own objective without considering the satisfactory levels of own decision variables and decreases it in favour of objectives at the lower level. Whereas, in the second approach, DM_0 does not specify the minimal satisfactory level for own objective, but instead DM_0 transfers the degree of satisfaction for not only own objective but also the own decision variables to the lower level. In both our approaches, with the help of analytic hierarchy process (AHP) method [Saaty TL. The analytical hierarchy process. New York: McGraw-Hill, 1980], DM_0 assigns weights $w_1, w_2, \ldots, w_k$ to objectives at the lower level. The most important idea to be emphasized is that equivalence is established such that the satisfactory levels of all objectives are proportional to their own weights. To obtain an overall satisfactory balance between both levels, by updating the satisfactory degree of the DM_0 which is in the first approach or the tolerances of the DM_0's decision variables which is in the second approach, transformed main problems are constructed corresponding to DTLLFP. Maximizing the least degree of equivalent satisfaction among all DMs, they efficiently find a satisfactory or compromise solution from a Pareto optimal set for DTLLFP problem. If the DM_0 is not satisfied with this solution, a strongly efficient satisfactory solution can be reached by interacting with him or her. An illustrative numerical example is provided to demonstrate the feasibility and efficiency of the proposed methods.

Keywords: Multiple level programming; Two-level linear fractional programming problem; Fuzzy programming; Fuzzy goals; Interactive methods; AHP

1. Introduction

Multiple level programming problems are frequently encountered in any hierarchical organization of large companies such as government agencies, profit or non-profit organizations, manufacturing plants, logistic companies, etc. Solution techniques explicitly assign each DM a unique objective, a set of decision variables, and a set of common constraints that affect all DMs. Each unit or department independently seeks its own interest, but is affected by the actions of other units.

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0305-0483/$ - see front matter
doi:10.1016/j.omega.2005.08.005

These problems have two interpretations depending on whether there is a cooperative relationship among DMs or not. For the first interpretation, the Stackelberg solution has been usually employed as a solution concept to multi-level programming problems [1–11]. These traditional approaches include vertex enumeration algorithms, approaches based on Karush–Kuhn–Tucker conditions and penalty functions [12]. They are effective only for solving very simple problems. In the simplest multi-level programming problem, a bi-level case, a decision maker at the upper level makes a decision subject to an optimization problem for a DM at the lower level. When the Stackelberg game [13]—a special case of a two-person, non-zero sum, non-cooperative game, with full information—is employed, it is assumed that there is no communication between the two DMs, or they do not make any binding agreement even if such communication exists. In the Stackelberg game, each DM completely knows objective functions and constraints of the two DMs, and the DM at the upper level (leader) first makes a decision and then the DM at the lower level (follower) specifies a decision so as to optimize an objective function with full knowledge of the decision of the leader. But, it is known that the problem for obtaining the Stackelberg solution is a special non-convex programming problem, even if the objective functions of both DMs and the common constraint functions are linear. So, the multi-level programming problems are difficult to solve and have been proved to be strongly NP-hard [14]. And also, in general, the Stackelberg solution does not satisfy Pareto optimality because of its non-cooperative nature.

For the second interpretation, concerning hierarchical decision problems in a decentralized firm, it is natural that decision makers are regarded to be cooperative rather than completely non-cooperative. To overcome the above-mentioned difficulties, for two-level linear programming problems, Lai [12] and Shih et al. [15] have proposed a solution concept which is different from the Stackelberg ones such that decisions of DMs at both levels are sequential and all of the DMs essentially cooperative with each other. Their method introduces a fuzzy approach for multi-level programming problem to simplify the complex nested structure by utilizing the concept of multi-objective decision making and the degree of satisfaction, in terms of fuzzy membership functions. And these in turn transfer the goals and decisions to a lower level in a top-down fashion for simulating the hierarchical decision making process. The so-called supervised search approach has been proven to be feasible and efficient. But, in several papers [16–22], Sakawa et al. emphasized that there is a possibility that Lai [12] and Shih et al. [15] methods lead a final solution to an undesirable one because of inconsistency between the fuzzy goals of the objective function and the decision variables. Because of additional constraints, the feasible region gets narrower or even may vanish. To avoid such problems in the methods of Lai [12] and Shih et al. [15], by eliminating the fuzzy goals for decision variables, Sakawa et al. [20] have developed interactive fuzzy programming for two-level linear programming problems. Moreover, from the viewpoint of experts' imprecise or fuzzy understanding of the nature of parameters in the problem formulation process, they have extended it to interactive fuzzy programming for two-level linear programming problems with fuzzy parameters [21]. These results have been extended to deal with two-level linear fractional programming problems both without and with fuzzy parameters [16,22]. Along these results, as additional results, extensions to multi-level 0–1 programming problems and two-level non-convex programming problems have been done through genetic algorithms [19,18]. On the other hand, in his paper, Shih [23] claimed that the interactive procedures for multi-level programming problems developed by Sakawa et al. [16–22] divert the difficulties of controlling decision variables and what is more, their procedures are similar to the interactive approaches in MODM.

In above-mentioned fuzzy approaches using the concept of tolerance membership functions and multi-objective decision making, the idea is to use the basic fuzziness and vagueness of such large hierarchical systems to make the complexity tractable. Generally, the upper level defines his or her tolerances by the use of membership function which constrains the lower level DMs' feasible space. The resulting iterative procedure, instead of the usually used extreme point search, relies on the change of membership functions that expresses the degree of satisfaction of the solutions to both the upper- and the lower-level DMs. So, these approaches explore the inherent vagueness of the system and thus generate no significant additional constraints. In fact, they significantly reduce the amount of computation required for large multiple level decentralized programming problems.

In this paper, we consider decentralized two-level linear fractional programming problems in which there are a single decision maker (DM_0) at the upper level and multiple DMs (DM_i, $i = 1, \ldots, k$) at the lower level, and objective functions of the DMs are linear fractional functions and constraint functions are linear functions. For this problem, we propose two new interactive approaches adopting both Sakawa et al. [16–22], and Lai [12], Shih et al. [15] opinions depending on whether additional restrictions via membership functions for decision variables are used. In our fuzzy interactive approach-I, by following Sakawa et al. [16–22], and also, thinking that for DMs it is more important the satisfactory levels of the objectives than those of decision variables, we did not use additional restrictions via