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Robust learning of automatic classes of languages ☆

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ABSTRACT

One of the most important paradigms in the inductive inference literature is that of robust learning. This paper adapts and investigates the paradigm of robust learning to learning languages from positive data. Broadening the scope of that paradigm is important: robustness captures a form of invariance of learnability under admissible transformations on the object of study; hence, it is a very desirable property. The key to defining robust learning of languages is to impose that the latter be automatic, that is, recognisable by a finite automaton. The invariance property used to capture robustness can then naturally be defined in terms of first-order definable operators, called translations. For several learning criteria amongst a selection of learning criteria investigated either in the literature on explanatory learning from positive data or in the literature on query learning, we characterise the classes of languages all of whose translations are learnable under that criterion.

1. Introduction

The present paper considers robust learning in the framework of inductive inference, more precisely, of Gold-style language learning in the limit. Informally, Gold [5] formalised language learning in the limit in a way that the learner is presented with all members, one at a time, of a language selected from a class of languages to be learnt. From this data, the learner has to identify the language in the limit by conjecturing and revising at most finitely often a hypothesis where the last hypothesis describes the language to be learnt correctly. Learning is robust when it is preserved under any admissible transformation of a learnable class: that is, given a learnable class, each of the images of the class under an admissible transformation, is learnable. Of course, the notion of an "admissible transformation" has to be appropriately and naturally defined. A related question is that of which classes of languages could be the object of learning, as the proposed "admissible transformations" will be defined with respect to those classes. This is a familiar theme, as the search for invariants is prominent in many fields of mathematics. For example, Hermann Weyl described Felix Klein's famous Erlangen programme on the algebraic foundation of geometry in these words [22]: "If you are to find deep properties of some object, consider all natural transformations that preserve your object." In the field of inductive inference, Bārzdiņš addressed the question of robust learning in the context of learning classes of recursive functions, and he conjectured the following, see [4,24]. Let a class of recursive functions be given. Then every image of the class under a general recursive operator is learnable iff the class is a subclass of a recursively enumerable (that is, a uniformly recursive) class of functions. To see where this conjecture

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comes from, it should be recalled that recursively enumerable classes of functions can be easily identified by a technique called "learning by enumeration" [5]. This technique amounts to simply conjecturing the first function in an effective list of the functions to learn, which is consistent with all data seen so far. The learnability of a class of functions by such an algorithm cannot be destroyed by transforming that class to another class using general recursive operators. So Bārzdiņš' conjecture essentially says that the enumeration technique fully captures robust learnability. Fulk [4] disproved the conjecture and this started a rich and fruitful exploration within the field of function learning [10,11,20]. Further refinements, such as uniform robust learnability [11] (where the learner for a transformed class has to be computable in a description of the transformation) and hyperrobust learnability [20] (learnability, by the same learner, of all transformations of a class under primitive recursive operators) have also been investigated.

It is natural to try and generalise robust learning to learning of classes of languages, first because the concept of robustness is an instance of the ubiquitous mathematical quest for invariants, and second because learning of classes of languages was the first object of study in inductive inference and has been more broadly investigated than function learning. However, what seems to be the natural extension of the definition from the context of function learning to the context of language learning does not work well, as even the class of singletons would not be robustly learnable according to the resulting definition. This paper proposes a modified approach to robust language learning, focusing on specific classes of languages, to be introduced in the next paragraph. Not only are these classes of languages well suited to the definition of a natural transformation between languages that can adequately capture a notion of robust learning and enjoy appealing characterisations; these classes of languages are also interesting in their own right and are themselves a new important topic of research. Besides the advantages of the restriction to those interesting languages, all concepts defined in this paper are meaningful even with respect to all r.e. languages.

Before we introduce the specific classes of languages which we have identified as the natural object of study for robust learning of languages, recall that sets of finite strings over some finite alphabet are regular if they are recognisable by a finite state automaton. Sets of pairs of finite strings over respective alphabets are regular if they are recognisable by a finite state multi-input automaton that uses two different inputs to read both coordinates of the pair, with a special symbol (say $\star$) being used to pad a shorter coordinate. For instance, to accept the pair $(010, 45)$ an automaton should read 0 from the first input and 4 from the second input and change its state from the start state to some state q_1, then read 1 from the first input and 5 from the second input and change its state from q_1 to some state q_2, finally read 0 from the first input and $\star$ from the second input and change its state from q_2 to an accepting state. It is essential that all inputs involved are read synchronically — one character per input and cycle. One can similarly consider finite state automata accepting triples, quadruples, and so on. The classes of languages that we focus on in this paper are classes of regular languages of the form $(L_i)_{i \in I}$ such that I and $\{(i, x){:}\ x \in L_i\}$ are regular sets; we refer to such a class as an automatic family of languages. An automatic family of languages is actually a particular kind of automatic structure, an object of study in its own right, which is now a source of many interesting questions and results on definability [6,14,15].

What this paper presents is not the first work to create a bridge between inductive inference and automatic structures: learnability of automatic families has recently been studied [7,8]. It should also be noted that our approach is an instance of a more general theme in inductive inference, that of the learnability of indexed families, a topic which has been extensively investigated in learning theory [1,17,18]: automatic families of languages are a special case of indexed families. One major advantage of automatic families over indexed families is that their first-order theory is decidable [6–8,14] and many of their important properties are first-order definable. In particular, the inclusion structure of an automatic family can be first-order defined. As we will see, this property plays an important role in this paper, and it is a key reason why robust learning can be fruitfully studied with automatic families.

With the right classes of languages in hand, we can then suitably define the admissible transformations of one class of languages into another that will capture a natural form of robust learning. We consider any transformation given by an operator Φ which maps sets of strings to sets of strings such that the automatic family $(L_i)_{i \in I}$ to be learnt is mapped to a family $(L'_i)_{i \in I} = (\Phi\langle L_i\rangle)_{i \in I}$, where Φ is definable by a first-order formula, Φ preserves inclusions amongst sets of strings, and Φ preserves noninclusions between members of the family. We call such a Φ a translator. A key result of the theory of automatic structures is that the image $(\Phi\langle L_i\rangle)_{i \in I}$ of an automatic family under such an operator Φ is again an automatic family [14]. An important special case is given by continuous, or text-preserving, translators for which $\Phi\langle L\rangle$ is the union of all $\Phi\langle F\rangle$ where F ranges over the finite subsets of L. Continuity is one of the most important properties in the general theory of functionals, and this work is no exception; it captures the natural requirement of computing more and more of the elements of the mapped language from larger and larger, but always finite, sets of elements of the original language. We study the impact of such translations on learnability.

We proceed as follows. In Sections 2 and 3, we introduce the necessary notation and concepts. In Section 4, we provide an overview of the main results to guide the reader in what comes next. In Section 5, we illustrate the notions with a few examples and provide a general characterisation of robust learnability in the limit of automatic families of languages. In Section 6 to 8, we provide many further characterisations of robust learnability for some of the learning criteria that have been studied in the literature: consistent and conservative learning, strong-monotonic learning, strong-monotonic consistent learning, finite learning. In Section 11, we consider learning from subset queries, learning from superset queries and learning from membership queries.