Journal of Computational Science 5 (2014) 685–695

Contents lists available at ScienceDirect

Journal of Computational Science

journal homepage: www.elsevier.com/locate/jocs

Dynamic analysis of pest control model with population dispersal in two patches and impulsive effect

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ARTICLE INFO

Article history:
Received 20 February 2014
Received in revised form 7 June 2014
Accepted 22 June 2014
Available online 9 July 2014

MSC:
34D23
92D30

Keywords:
Impulsive differential equations
Global stability
Floquet theorem
Pest control

ABSTRACT

In this paper, we investigate the pest control model with population dispersal in two patches and impulsive effect. By exploiting the Floquet theory of impulsive differential equation and small amplitude perturbation skills, we can obtain that the susceptible pest eradication periodic solution is globally asymptotically stable if the impulsive periodic τ is less than the critical value τ_0. Further, we also prove that the system is permanent when the impulsive periodic τ is larger than the critical value τ_0. Hence, in order to drive the susceptible pest to extinction, we can take impulsive control strategy such that $\tau < \tau_0$ according to the effect of the viruses on the environment and the cost of the releasing pest infected in a laboratory. Finally, numerical simulations validate the obtained theoretical results for the pest control model with population dispersal in two patches and impulsive effect.

1. Introduction

It is well known that the traditional method for the pest control depends on the seasonal use of chemical pesticides, while it has been found that the unrestrained use of persistent pesticide not only can increase the incidence of pesticide-resistant pest varieties, but also can inflict harmful effects on humans through the accumulation of hazardous chemicals in their food chain [1–4]. Moreover, pesticide pollution is also a major threat to beneficial insects, and these insects are sometimes more affected by pesticide spraying than target pests. In fact, the use of chemical pesticides will generate many deleterious effects, which need to be reduced or eliminated. Besides, the effectiveness of chemical pesticides decreases, which occurs to the adaptation of the pests to such pesticides, and it also leads to an exponential increase in the cost of spraying chemicals on pests [5–7]. As a result, many researchers start placing much emphasis on tactics other than chemical controls, including the deployment of crop varieties that are resistant to pests, genetic, cultural and biological methods, more and more experts have the intense interest in studying biological control, and investigate the biological and cultural methods of pest control that are ecologically feasible and can give the answers that are sustainable in the long term [8–14]. Andson and May [15,16] have studied the dynamics of inset–pathogen interactions.

In fact, the use of biologically based technologies for pest control is a very effective method in the integrated pest management, which is also widely payed more attention to in ecologists and applied mathematicians. One of the important and effective biological controlling methods is by means of releasing the natural enemies, which plays an important role in suppressing insect pests. However, the cultivation of the natural enemy in laboratories is very difficult, and the cost is also very high in general. Especially, when encountering the situation of some disaster, which is limited in a small range, the method is not very effective or ideal at this time. Moreover, the researchers have made lots of important progress in biological control, theoretically as well as experimentally [17–21].

Many researchers have widely studied the dynamics of population dispersal in multiple different patches [22–28]. Xiang and Song [29] have studied the dynamic behaviors of a two-prey two-predator system with impulsive effect on the predator of fixed moment. Some researchers have controlled the pests by using viruses and simultaneously releasing the pest population [30,31]. First, a small amount of pathogens is introduced into

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http://dx.doi.org/10.1016/j.jocs.2014.06.011
1877-7503/

a pest population with the expectation that it will generate an epidemic and that it will subsequently be endemic. The success of this approach depends on the survival of the microbes which in turn depends on environmental factors. At the same time, we consider to release the pests infected in the laboratories to the pest population with periodic impulsive effect. The infected pest has little effect on the crops. The susceptible pests become infected through direct contact with the infective ones or through encountering with the free-living infective stage in the environment. Thus it can infect the pest population and result in the death of them. The main purpose of this paper is to formulate and investigate an epidemiological model for the bio-control of a pest. In fact, the theoretical investigation and its application analysis can be found in almost every field [32–40]. This pest population is assumed to grow according to a logistic curve in the absence of disease [41,42]. In this paper, we assume that the spread of disease of the pest only happens in two patches, and the dispersal only happens at the impulse time, when some infected pests are introduced. Applying the comparison theorem of impulsive differential equation, we can obtain the sufficient conditions for global asymptotical stability of semi-trivial periodic solution and the permanence of system.

The paper is organized as follows: In Section 2, we establish the pest model and introduce some definitions. In Section 3, we give some lemmas and theorems. We analyze the conditions for the globally asymptotical stability and the permanence of the semi-trivial periodic solution. In Section 4, numerical simulations are given to illustrate the feasibility of our results.

2. Model formulation

Let $N_i(t)$ $(i=1, 2)$ denotes the density of an original insect pest population in the ith $(i=1, 2)$ patch. Suppose that the style of their growth conforms to the Logistic curve, the maximum capacity of two environment are $r_1/a_1, r_2/a_2$, respectively. Their intrinsic birthrate are r_1, r_2, respectively. Then we can obtain the dynamics of $N_1(t)$ and $N_2(t)$ as the following differential equation, respectively, by establishing mathematical model and considering the practical value:

$$\begin{cases} \dot{N}_1(t)=N_1(t)(r_1-a_1N_1(t)), \\ \dot{N}_2(t)=N_2(t)(r_2-a_2N_2(t)). \end{cases}$$

When a pest pathogen as biotic insecticide intrudes into the pest community, the pest species is divided into two classes: The first class is the susceptible pest whose density are represented by $S_i(t)(i=1, 2)$ at the time t in the ith $(i=1, 2)$ patch; the second class is the infected pest whose density is denoted by $I_i(t)$ $(i=1, 2)$ at the time t in the ith $(i=1, 2)$ patch. So the total density of the population at any time t is

$$\begin{cases} N_1(t)=S_1(t)+I_1(t), \\ N_2(t)=S_2(t)+I_2(t). \end{cases}$$

We further assume that the pest cannot carry on the dispersal in two different patches, and the susceptible pest individuals are capable of reproducing. The incidence of the pest disease is bilinear incidence. The incidence is given by the simple mass action incidence with transmission coefficient $\lambda_i>0(i=1, 2)$ in the ith $(i=1, 2)$ patch. The constant $\beta_i>0(i=1, 2)$ in the ith $(i=1, 2)$ patch act as the mortality due to the illness. Thus we can obtain the insect–pathogen model as follows:

$$\begin{cases} \dot{S}_1(t)=(r_1-a_1S_1(t))S_1(t)-\lambda_1S_1(t)I_1(t), \\ \dot{S}_2(t)=(r_2-a_2S_2(t))S_2(t)-\lambda_2S_2(t)I_2(t), \\ \dot{I}_1(t)=\lambda_1S_1(t)I_1(t)-\beta_1I_1(t), \\ \dot{I}_2(t)=\lambda_2S_2(t)I_2(t)-\beta_2I_2(t). \end{cases} \tag{2.1}$$

If the pests have the dispersal in two patches, $a_{ii}>0$ $(i=1, 2)$ represents the migration coefficient of the susceptible pest in the ith $(i=1, 2)$ patch, and $b_{ii}>0(i=1, 2)$ represents the migration coefficient of infected pest in the ith $(i=1, 2)$ patch, then the model (2.1) can be rewritten as

$$\begin{cases} \dot{S}_1(t)=(r_1-a_1S_1(t))S_1(t)-\lambda_1S_1(t)I_1(t)-a_{11}S_1(t)+a_{22}S_2(t), \\ \dot{S}_2(t)=(r_2-a_2S_2(t))S_2(t)-\lambda_2S_2(t)I_2(t)-a_{22}S_2(t)+a_{11}S_1(t), \\ \dot{I}_1(t)=\lambda_1S_1(t)I_1(t)-\beta_1I_1(t)-b_{11}I_1(t)+b_{22}I_2(t), \\ \dot{I}_2(t)=\lambda_2S_2(t)I_2(t)-\beta_2I_2(t)-b_{22}I_2(t)+b_{11}I_1(t). \end{cases} \tag{2.2}$$

If the infected pest are introduced, the model (2.2) can be given as

$$\begin{cases} \dot{S}_1(t)=(r_1-a_1S_1(t))S_1(t)-\lambda_1S_1(t)I_1(t)-a_{11}S_1(t)+a_{22}S_2(t), \\ \dot{S}_2(t)=(r_2-a_2S_2(t))S_2(t)-\lambda_2S_2(t)I_2(t)-a_{22}S_2(t)+a_{11}S_1(t), \\ \dot{I}_1(t)=\lambda_1S_1(t)I_1(t)-\beta_1I_1(t)-b_{11}I_1(t)+b_{22}I_2(t)+\alpha_1, \\ \dot{I}_2(t)=\lambda_2S_2(t)I_2(t)-\beta_2I_2(t)-b_{22}I_2(t)+b_{11}I_1(t)+\alpha_2, \end{cases} \tag{2.3}$$

where $\alpha_i>0\,(i=1,2)$ represents the release amount of infected pests at the ith patch.

If we introduce the pest pathogen and infected pests into the model (2.3) at the impulsive time, and the patch dispersal only happens between susceptible pest individuals at just the impulsive time, thus we can obtain the following biologic control model:

$$\begin{cases} \left.\begin{array}{l} \dot{S}_1(t)=(r_1-a_1S_1(t))S_1(t)-\lambda_1S_1(t)I_1(t), \\ \dot{S}_2(t)=(r_2-a_2S_2(t))S_2(t)-\lambda_2S_2(t)I_2(t), \\ \dot{I}_1(t)=\lambda_1S_1(t)I_1(t)-\beta_1I_1(t), \\ \dot{I}_2(t)=\lambda_2S_2(t)I_2(t)-\beta_2I_2(t), \end{array}\right\} & t\neq n\tau, \\ \left.\begin{array}{l} \Delta S_1(t)=-u_1S_1(t)-a_{11}S_1(t)+a_{22}S_2(t), \\ \Delta S_2(t)=-u_2S_2(t)-a_{22}S_2(t)+a_{11}S_1(t), \\ \Delta I_1(t)=u_1S_1(t)+\alpha_1, \\ \Delta I_2(t)=u_2S_2(t)+\alpha_2, \end{array}\right\} & t=n\tau, \end{cases} \tag{2.4}$$

where

$$u_i>0, a_i>0, 0<u_i+a_{ii}<1, \quad i=1,2,$$

$$\Delta S_i(t)=S_i(t^+)-S_i(t), \Delta I_i(t)=I_i(t^+)-I_i(t), \quad i=1,2,$$

u_i denotes the probability that the pest is infected due to the pest pathogen at $t=n\tau$ in the ith patch, where τ represents the impulsive cycle.