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JOCS-130; No. of Pages 10
ARTICLE IN PRESS

Journal of Computational Science xxx (2012) xxx–xxx

Contents lists available at SciVerse ScienceDirect

Journal of Computational Science

journal homepage: www.elsevier.com/locate/jocs

Fuzzy stochastic inequality and equality possibility constraints and their application in a production-inventory model via optimal control method

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ARTICLE INFO

Article history:
Received 15 August 2011
Received in revised form 11 March 2012
Accepted 16 March 2012
Available online xxx

Keywords:
Fuzzy equality
Interval number
Fuzzy stochastic inequality
Bang–Bang control

ABSTRACT

This paper deals with one equality constraint in fuzzy environment and other inequality constraint with both fuzzy and random parameter together. The purpose of this paper is to demonstrate the application of these type of constraints in a production inventory model solved as a Bang–Bang control problem in a finite time horizon. Finally numerical experiments have been performed for illustration.

1. Introduction

Since the publication of Zadeh's paper [26] in 1965, fuzzy set theory has become a powerful technique for dealing with nonstochastic imprecision and vagueness. Analogous to chance constrained programming with stochastic parameters, in a fuzzy environment, it is assumed that some constraints are satisfied with a least possibility, η [here the 'chance' is represented by the 'possibility']. Zadeh [27] and Dubois and Prade [5] introduced the possibility constraints which are very relevant to the real-life decision making problems and presented the process of defuzzification for these constraints. Later, Liu and Iwamura [13,14] have extended these ideas and applied in a linear/nonlinear programming problem. Maiti and Maiti [16] also applied this concept in a two warehouse inventory problem. But till now, only few researchers considered the equality or inequality relation in fuzzy sense and applied it in decision making problems. There are few articles [10,23,15] which deal with both fuzzy and random data. The purpose of the present paper is to introduce the fuzzy equality and fuzzy random inequality constraints in a real-world multi-item production-inventory problem formulated as an control problem.

Moreover, in decision making problems, some fuzzy coefficients/parameters may occur in a objective function. In order to handle this kind of coefficients or parameters, these quantities are represented by fuzzy numbers and following Grzegorzewski [8] are approximated to crisp sets of interval numbers.

One of the weaknesses of some production-inventory models is the unrealistic assumption that all items produced are of good quality. But production of defective units is a natural phenomenon in a production process. Defective items as a result of imperfect quality production process were initially considered by Porteus [22] and later by several researchers such as Salameh and Jaber [24], Goyal and Cardenas-Barron [7], Wee et al. [25], Jaber and Bonney [12], Minner and Kleber [20] etc.

In production models, normally it is assumed that production is continued up to a certain period of time and an inventory is built up by that time. Once sufficient stock level is reached, the production is stopped and demand is met from the accumulated stock till the stock is exhausted or some shortages are allowed. After this (i.e. after a certain period of time), production is again commenced. This is also somewhat unrealistic. Optimal control formulation is most suitable for the formulation of finite time horizon production models. It allows dynamic state and control variables for the system during a finite period of time. Very often, in inventory and production system, production and stock level(control variables) are dependent on demand at that time. Optimal control formulation and solution via optimal control theory are most appropriate for the formulation of real-life production and inventory problems in dynamic environment. But only a very

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doi:10.1016/j.jocs.2012.03.005

few researchers (cf. Axsater and Rosling [1], Cho [4], and others) have applied optimal control theory to inventory control system. Moreover, till now, those who have adopted the optimal control theory have formulated the inventory control problems in crisp environment only. None has tried this in other environments – i.e. fuzzy and fuzzy-stochastic environments. This vacuum and the suitability of the optimal control theory motivated us to use it in the formulation and solution of the present inventory control problems in a different environment – fuzzy environment. Optimal control theory is an extension of the calculus of variations problems. Bellman applied optimal control theory for dynamic programming problems (cf. Bellman and Kalaba [2], Hull, [9]). Maity and Maiti applied this conception in their models [17–19]. Again, control of the most of the system (e.g., engineering, social, political, etc.) tends to oscillate between two extremities, i.e., controlled by Bang–Bang control process which is more faster than proportional control.

In this paper, the measures of fuzzy equality in possibility sense, inequality for fuzzy and random quantities in probability sense following Liu and Iwamura [13] and Luhandjula [15] are defined. The respective fuzzy numbers are represented into corresponding crisp intervals following Grzegorzewski [8]. For illustration we investigate an economic production quantity (EPQ) model with imprecise total budget and uniformly random available space. The constraints are taken in equality and inequality sense respectively in fuzzy and fuzzy-stochastic environment. The demand is deterministic and depends on the stock. The relevant inventory costs due to production, holding and shortage (if allowed) are taken in fuzzy sense. The total cost is minimized formulating the problem as an optimal control problem. It is solved following the Pontryagin's principle (cf. Pontryagin and Boltyanski [21]), the Kuhn–Tucker conditions and generalized reduced gradient(GRG) technique (cf. Gabriel and Ragsdell [6]). The optimum production and stock levels are determined for known demand function. The model is illustrated through numerical examples and results are presented graphically also.

2. Preliminary

2.1. Fuzzy set theory

Fuzzy set is a class of objects in which there is no sharp boundary between those objects that belong to the class and those that do not. Let X be a collection of objects and x be an element of X, then a fuzzy set $\widetilde{A}$ in X is a set of ordered pairs $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x))/x \in X\}$, where $\mu_{\widetilde{A}}(x)$ is called the membership function or grade of membership of x in $\tilde{A}$ which maps X to the membership space M which is considered as the closed interval $[0, u]$, where $0 < u \leq 1$.

Normality: A fuzzy set $\widetilde{A}$ is normal if there exist at least one element $x \in X$, such that $\mu_{\widetilde{A}}(x) = 1$.

α**-Level set:** α-level set of a fuzzy number $\widetilde{A}$ in X is denoted by A_α and is defined as, 'the set of elements which belongs to the fuzzy set $\widetilde{A}$ at least to the degree α', i.e., $A_\alpha = \{x \in X/\mu_{\widetilde{A}}(x) \geq \alpha\}$ and $A'_\alpha = \{x \in X/\mu_{\widetilde{A}}(x) > \alpha\}$ is called a "strong α-level set".

Convexity: A fuzzy set $\widetilde{A}$ in X is said to be convex if and only if for any $x_1, x_2 \in X$, the membership function of $\widetilde{A}$ satisfies the inequality $\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min\{\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)\}$ for $0 \leq \lambda \leq 1$.

Fuzzy number: A fuzzy number is a special class of a fuzzy sets. Different definitions and properties of fuzzy numbers are encountered in the literature but they all agree on that a fuzzy number represents the conception of "a set of real numbers close to a", where 'a' is the number being fuzzyfied.

A fuzzy number is a fuzzy set in the universe of discourse X that is both convex and normal. The term "fuzzy number", is used to handle imprecise numerical quantities. For example, shortage cost of a commodity is about \$5. A general definition of a fuzzy number is a real fuzzy number $\tilde{A}$ described as a fuzzy subset on the real line $\Re$ whose membership function $\mu_{\tilde{A}}(x)$ is

(i) a continuous mapping from $\Re$ to the closed interval [0,1],
(ii) constant on $(-\infty, a_1]$: $\mu_{\tilde{A}}(x) = 0$, $\forall x \in (-\infty, a_1]$,
(iii) strictly increasing on $[a_1, a_2]$: e.g., $\mu_{\tilde{A}}(x) = f(x)$, $\forall x \in [a_1, a_2]$ where $f(x)$ is a strictly increasing function of x,
(iv) constant on $[a_2, a_3]$: e.g., $\mu_{\tilde{A}}(x) = 1$, $\forall x \in [a_2, a_3]$,
(v) strictly decreasing on $[a_3, a_4]$, e.g., $\mu_{\tilde{A}}(x) = g(x)$, $\forall x \in [a_3, a_4]$ where $g(x)$ is a strictly decreasing function of x,
(vi) constant on $[a_4, \infty)$: e.g., $\mu_{\tilde{A}}(x) = 0$, $\forall x \in [a_4, \infty)$.

2.2. Interval arithmetic

Throughout this section lower case letters denote real numbers and upper case letter denote closed intervals. The set of all positive real numbers is denoted by R^+. An order pair of brackets defines an interval $A = [a_L, a_R] = \{a : a_L \leq a \leq a_R, a \in R^+\}$ where a_L and a_R are respectively left and right limits of A.

2.2.1. Operations

Let $* \in \{+, -, \cdot, /\}$ be a binary operation on the set of positive real numbers. If A and B are closed intervals then $A * B = \{a * b : a \in A, b \in B\}$ defines a binary operation on the set of closed intervals. In the case of division, it is assumed that $0 \notin B$. The operations on intervals used in this paper may be explicitly calculated from the above definition as

$$\begin{aligned} \frac{A}{B} &= \frac{[a_L, a_R]}{[b_L, b_R]} = \left[\frac{a_L}{b_R}, \frac{a_R}{b_L}\right] \text{ where } 0 \notin B,\ 0 \leq a_L \leq a_R \text{ and } 0 < b_L \leq b_R \\ A + B &= [a_L, a_R] + [b_L, b_R] = [a_L + b_L, a_R + b_R] \\ kA &= \begin{cases} [ka_L, ka_R], & \text{for } k \geq 0 \\ [ka_R, ka_L], & \text{for } k < 0 \quad \text{for a real number } k \end{cases} \end{aligned} \tag{1}$$

Please cite this article in press as: B. Das, M. Maiti, Fuzzy stochastic inequality and equality possibility constraints and their application in a production-inventory model via optimal control method, J. Comput. Sci. (2012), doi:10.1016/j.jocs.2012.03.005