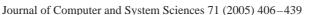


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## A complete and equal computational complexity classification of compaction and retraction to all graphs with at most four vertices and some general results $\stackrel{\text{$\stackrel{\frown}{$}}}{\sim}$

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Received 15 December 2003

Available online 23 May 2005

## Abstract

A very close relationship between the compaction, retraction, and constraint satisfaction problems has been established earlier providing evidence that it is likely to be difficult to give a complete computational complexity classification of the compaction and retraction problems for reflexive or bipartite graphs. In this paper, we give a complete computational complexity classification of the compaction and retraction and retraction problems for all graphs (including partially reflexive graphs) with four or fewer vertices. The complexity classification of both the compaction and retraction problems is found to be the same for each of these graphs. This relates to a long-standing open problem concerning the equivalence of the compaction and retraction problems. The study of the compaction and retraction problems for graphs with at most four vertices has a special interest as it covers a popular open problem in relation to the general open problem. We also give complexity results for some general graphs. The compaction and retraction problems are special graph colouring problems, and can also be viewed as partition problems with certain properties. We describe some practical applications also.

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Keywords: Computational complexity; Graph; Colouring; Homomorphism; Retraction; Compaction; Partition

 $<sup>^{\</sup>diamond}$  A version presenting the results of this paper appears in Proceedings of the Tenth Annual International Computing and Combinatorics Conference (COCOON), 2004.

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## 1. Introduction

A version presenting the results of this paper appears in [29]. We first introduce the following definitions and problems, and then describe the motivation and results.

## 1.1. Definitions

The pair of vertices forming an edge in a graph are called the *endpoints* of the edge. An edge vv with the same endpoints in a graph is called a *loop*, and the vertex v is said to have a loop. A *reflexive graph* is a graph in which every vertex has a loop. An *irreflexive graph* is a graph which has no loops. Any graph, in general, is a partially reflexive graph, meaning that its individual vertices may or may not have loops. Thus reflexive and irreflexive graphs are special partially reflexive graphs. A *bipartite graph G* is a graph whose vertex set can be partitioned into two distinct subsets  $G_A$  and  $G_B$ , such that each edge of G has one endpoint in  $G_A$  and the other endpoint in  $G_B$ ; we say that  $(G_A, G_B)$  is a bipartition of G. Thus a bipartite graph is irreflexive by definition. When we do not mention the terms reflexive, irreflexive, or bipartite, the corresponding graph may be assumed to be a partially reflexive graph. An edge is said to be *incident* with a vertex v in a graph if v is an endpoint of the edge. A vertex u is said to be *adjacent* to a vertex v in a graph if uv is an edge of the graph; if u is adjacent to v then v is also adjacent to u. If a vertex u is adjacent to a vertex v in a graph then u is said to be a *neighbour* of v, and v is said to be a neighbour of u in the graph. The *neighbourhood* of a vertex v in a graph, denoted as Nbr(v), is the set of all neighbours of v in the graph (note that for a loop vv, we have  $v \in Nbr(v)$ ). A graph in which each pair of distinct vertices are adjacent is called a *complete graph*. We denote an irreflexive complete graph with k vertices by  $K_k$ . A bipartite graph, with bipartition  $(G_A, G_B)$ , in which every vertex in  $G_A$  is adjacent to every vertex in  $G_B$  is called a *complete bipartite graph*. A path of length k - 1 is a graph containing k distinct vertices, say  $v_0, v_1, v_2, \ldots, v_{k-1}$ , such that  $v_0v_1, v_1v_2, \ldots, v_{k-2}v_{k-1}$  are all the non-loop edges of the graph,  $k \ge 1$ ; we may write such a path as  $v_0 v_1 v_2 \dots v_{k-1}$ , where the vertex  $v_0$  is called the *origin* or the *first vertex* of the path, the vertex  $v_{k-1}$  is called the *terminus* or the *last vertex* of the path, and the vertices  $v_1, v_2, \ldots, v_{k-1}$  are called the *internal vertices* of the path. A cycle of length k, called a k-cycle, is a graph containing k distinct vertices, say  $v_0, v_1, v_2, \ldots, v_{k-1}$ , such that  $v_0v_1, v_1v_2, \ldots, v_{k-2}v_{k-1}, v_{k-1}v_0$  are all the non-loop edges of the graph,  $k \ge 3$ ; we may write such a cycle as  $v_0v_1v_2 \dots v_{k-1}v_0$ . A square will be used as a synonym for a 4-cycle. A *triangle* will be used as a synonym for a 3-cycle. A *walk* of length n in a graph is a sequence of vertices  $v_0 v_1 v_2 \dots v_n$  not necessarily distinct such that  $v_i v_{i+1}$  is an edge of the graph, for all  $i = 0, 1, 2, ..., n - 1, n \ge 0$ ; we say that such a walk is from  $v_0$  to  $v_n$ . For a graph G, we use V(G) and E(G) to denote the vertex set and the edge set of G respectively. The size of a graph is the number of vertices plus the number of edges in the graph. We define min S and max S to give the minimum and the maximum element respectively in a set S. When a set S is an argument of a mapping f, we define  $f(S) = \{f(s) | s \in S\}$ . If a set has only one vertex, we may just write the vertex instead of the set.

Let G be a graph. A vertex v of G is said to be an *isolated vertex* of G, if v is not adjacent to any other vertex v' of G,  $v \neq v'$  (note that an isolated vertex may have a loop). Two vertices u and v of G are said to be *connected* in G, if there exists a path from u to v in G; otherwise u and v are said to be *disconnected* in G. The *distance* between a pair of vertices u and v in G, denoted as  $d_G(u, v)$  or  $d_G(v, u)$ , is the length of a shortest path from u to v in G, if u and v are connected in G; we define  $d_G(u, v)$ (and  $d_G(v, u)$ ) to be infinite, if u and v are disconnected in G. The *diameter* of G is the maximum Download English Version:

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