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The pessimistic diagnosabilities of some general regular graphs

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A R T I C L E I N F O

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A B S T R A C T

The pessimistic diagnosis strategy is a classic strategy based on the PMC model. A system is t/t-diagnosable if, provided the number of faulty processors is bounded by t, all faulty processors can be isolated within a set of size at most t with at most one fault-free node mistaken as a faulty one. The pessimistic diagnosability of a system G, denoted by $t_p(G)$, is the maximal number of faulty processors so that the system G is t/t-diagnosable. In this paper, we study the pessimistic diagnosabilities of some general k-regular k-connected graphs G_n. The main result $t_p(G_n) = 2k - 2 - g$ under some conditions is obtained, where g is the maximum number of common neighbors between any two adjacent vertices in G_n. As applications of the main result, every pessimistic diagnosability of many famous networks including some known results, such as the alternating group networks AN_n, the k-ary n-cubes Q_n^k, the star graphs S_n, the matching composition networks $G(G_1, G_2; M)$ and the alternating group graphs AG_n, are obtained.

1. Introduction

A system or a network is modeled as an undirected graph with each processor represented as a node and each communication link represented as an undirected edge. In a multiprocessor system, some of these nodes may be faulty when the system is put into use. As the number of nodes in a multiprocessor system increases, identifying faulty nodes is crucial for reliable computing. The process of identifying faulty processors is called the diagnosis of the system. When a faulty node is identified, it is replaced by a fault-free node to maintain the system's reliability. An interconnection network's diagnosability is an important measure of its self-diagnostic capability. A system is said to be *t-diagnosable* if all faulty units can be identified provided the number of faulty units present does not exceed t. The diagnosability of a system is the maximal number of faulty processors that the system can guarantee to diagnose.

A number of models has been proposed for diagnosing faulty processors in a network. Preparata et al. [21] first introduced a graph theoretical model, the so-called *PMC model* (i.e., Preparata, Metze and Chien's model), for system level diagnosis in multiprocessor systems. In this model, it is assumed that a processor can test the faulty or fault-free status of another processor. It is assumed that if a vertex is fault-free it should always give correct and reliable test results and if a vertex is faulty then its test result may be correct or incorrect. The pessimistic diagnosis strategy is a classic strategy based on the PMC model in which isolates all faulty vertices within a set containing at most one fault-free vertex. A system is *t/t-diagnosable*, provided the number of faulty processors is bounded by t, all faulty processors can be isolated within a set

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of size at most t with at most one fault-free node mistaken as a faulty one. The *pessimistic diagnosability* of a system G, denoted by $t_p(G)$, is the maximal number of faulty processors so that the system G is t/t-diagnosable. The pessimistic diagnosabilities of many interconnection networks have been explored, as examples to see [3–7,19,23,24,28,29,32] etc. The pessimistic diagnosability of alternating group graphs AG_n and the hypercube-like networks (BC graphs) were obtained by Tsai in [25] and [26], respectively. Let G be a graph, $w \in V(G)$ is called a *common neighbor* of u and v in G if w is adjacent to both u and w in G.

In this paper, we study the pessimistic diagnosabilities of some general k-regular k-connected graphs G_n under the PMC model by applying the method used in [25]. The main result $t_p(G_n) = 2k - 2 - g$ under some conditions is obtained, where g is the maximum number of common neighbors between any two adjacent vertices in G_n. As applications and corollaries of the main result, every pessimistic diagnosability of many famous networks, such as the alternating group networks AN_n, the k-ary n-cubes Q_n^k, the star graphs S_n, the matching composition networks $G(G_1, G_2; M)$ and the alternating group graphs AG_n, are obtained.

The rest of this paper is organized as follows. Section 2 introduces some definitions and notations. Section 3 is devoted to the pessimistic diagnosability of the regular graph G_n, the main result is derived. Section 4 concentrates on the applications to some famous networks. Section 5 concludes the paper.

2. Preliminaries

In this section, we give some terminologies and notations of combinatorial network theory. For terminologies and notations not defined here, the reader is referred to [2].

2.1. Terminologies and notations

We use a graph, denoted by $G = (V(G), E(G))$, to represent an interconnection network, where $V(G)$ is the vertex set of G; $E(G)$ is the edge set of G. Here, a vertex $u \in V(G)$ represents a processor and an edge $(u, v) \in E(G)$ represents a link between vertices u and v. For a vertex $u \in V(G)$, we use the symbol $N_G(u)$ to denote a set of vertices in G adjacent to u. For a vertex set $U \subseteq V(G)$, let $N_G(U) = \bigcup_{v \in U} N_G(v) - U$ and $G[U]$ be the subgraph of G induced by U. If $|N_G(u)| = k$ for any vertex in G, then G is *k-regular*. Let G be a connected graph, if $G - S$ is still connected for any $S \subseteq V(G)$ with $|S| \leq k - 1$, then G is *k-connected. For any two vertices u and v in G, let* $cn(G; u, v)$ denote the number of vertices who are the neighbors of both u and v, that is, $cn(G; u, v) = |N_G(u) \cap N_G(v)|$. Let $cn(G) = \max\{cn(G; u, v) : u, v \in V(G)\}$. Let $|V(G)|$ be the size of vertex set and $|E(G)|$ be the size of edge set. A graph which contains no loops and no parallel edges is *simple* Throughout this paper, all graphs are finite, undirected simple graphs.

Let $[n] = \{0, 1, 2, \ldots, n-1\}$ (not the general notation $[n] = \{1, 2, \ldots, n\}$). For a finite group A and a subset S of A such that $1 \notin S$ and $S = S^{-1}$ (where 1 is the identity element of A), the *Cayley graph* $\mathrm{Cay}(A; S)$ on A with respect to S is defined to have vertex set A and edge set $\{(g, gs) | g \in A, s \in S\}$.

Definition 1. Let $n, s, r \geq 0$ and $p, m \geq 1$ be integers. An *n-th regular graph*, say G_n, can be recursively constructed as follows:

(1) 1-th regular graph, say G_1, is a r-connected r-regular simple graph with order p.
(2) For $n \geq 2$, n-th regular graph, say G_n, is a regular graph which consists of m_n copies of $(n-1)$-th regular graph G_{n-1}, say $G_{n-1}^0, G_{n-1}^1, \ldots, G_{n-1}^{m_n-1}$. Each vertex $v \in G_{n-1}^i$ has s $(1 \leq s < m_n)$ neighbors, called extra neighbors of v, outside G_{n-1}^i for every $i \in [m_n]$. Let the set of extra neighbors of $v \in V(G_{n-1}^i)$ for any $i \in [n]$ be $N_e(v)$ such that $0 \leq |N_e(v)| \leq 1$.
(3) G_n is $[(n-1)s + r]$-regular and $[(n-1)s + r]$-connected.

It is easily verified that the order of G_n is $N = m_2 m_3 \ldots m_n p$.

2.2. Some interconnection networks regarded as special regular graphs G_n

2.2.1. The alternating group graph AG_n

Jwo et al. [18] introduced the alternating group graph as an interconnection network topology for computing systems.

Definition 2. Let A_n be the alternating group of degree n with $n \geq 3$. Set $S = \{(1\ 2\ i), (1\ i\ 2) \mid 3 \leq i \leq n\}$. The *alternating group graph*, denoted by AG_n, is defined as the Cayley graph $AG_n = \mathrm{Cay}(A_n, S)$.

AG_3 is a triangle. It is clear that AG_n is a $(2n-4)$-connected and $(2n-4)$-regular graph with $n!/2$ vertices. AG_n can be divided into n sub-alternating group graphs $AG_n^0, AG_n^1, \ldots, AG_n^{n-1}$. For each $i \in [n]$, AG_n^i is isomorphic to AG_{n-1}. For each vertex $v \in AG_n^i$, v has exactly two neighbors that are not contained in AG_n^i, which are called the *extra neighbors* of v.

Lemma 1. *(See [15].) The extra neighbors of every vertex of AG_n are in different subgraphs AG_n^i for $n \geq 4$. For any two different vertices u, v, $cn(AG_n : u, v) = 1$ if u and v are adjacent; otherwise, $cn(AG_n : u, v) \leq 2$.*