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On conditional fault tolerance and diagnosability of hierarchical cubic networks

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ABSTRACT

Fault tolerance is especially important for interconnection networks, since the growing size of networks increases their vulnerability to component failures. A classical measure for the fault tolerance of a network in the case of vertex failures is its connectivity. Given a network based on a graph G and a positive integer h, the R^h-connectivity of G is the minimum cardinality of a set of vertices in G, if any, whose deletion disconnects G, and the minimum degree of every connected component is at least h. This paper investigates the R^h-connectivity ($h = 1, 2$) of the hierarchical cubic network HCN_n ($n \geq 2$), and shows that $\kappa^1(HCN_n) = 2n$, $\kappa^2(HCN_n) = 4n - 4$, respectively. Furthermore, the paper establishes the conditional diagnosability of HCN_n under the PMC diagnostic model.

1. Introduction

Many large-scale multiprocessor or multicomputer systems take interconnection networks as underlying topologies. The scale of some supercomputers has been amplifying dramatically recently. For example, the Tianhe-2 contains 3,120,000 nodes and the Cray Titan contains 560,640 nodes [38]. As the number of processors in such systems increases, processor failure is inevitable. To ensure the stable running of the systems, we must find out the faulty processors to repair or replace them. *System-level diagnosis*, as a powerful tool, has been widely used in VLSI as well as wire and wireless networks. The field of system-level diagnosis has evolved from the pioneering work of Preparata, Metze, and Chien [19], who proposed the first diagnostic model, known as PMC model. The PMC model assumed that each node can test its neighboring nodes, and the test results are "faulty" or "fault-free". Under this model, the diagnosability of an interconnection network is the maximum number of faulty nodes in the system that can be guaranteed to be located. To grant more accurate measurement of diagnosability for a large-scale processing system, Lai et al. [16] introduced the conditional diagnosability of a system under the PMC model, by assuming that the probability that all adjacent nodes of one node are faulty simultaneously is very small. That is to say, conditional diagnosability is the diagnosability under the condition that all adjacent nodes of any node cannot be faulty simultaneously. They further showed that the conditional diagnosability of Q_n is $4(n-2)+1$ for $n \geq 5$. Since then, the conditional diagnosabilities of some variants of hypercube, such as matching composition networks [26], folded hypercubes [36], k-ary n-cubes [5], shuffle cubes [25], augmented cubes [3], balanced hypercubes [27], and dual cubes [29], have been established. As for more complex networks, such as *DCC* linear congruential graphs [10], (n,k)-arrangement

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graphs [18], alternating group networks [17], and star graphs [4], their conditional diagnosabilities under the PMC model have also been derived. Some generalizations [20,31] and some unified approaches [9,15,37] to measure the conditional diagnosability have been explored. Recently, Tsai [21], and Teng et al. [22] have made progress in design of diagnosis algorithm under the PMC model.

This paper focuses on the conditional diagnosability of the hierarchical cubic networks, proposed by Ghose and Desai [14] as a hypercube-based topology while preserving its attractive features [2,7,11,12,14,32,33].

The rest of this paper is organized as follows. In Section 2, we recall some definitions, notations and the structure of the n-dimensional hierarchical cubic network HCN_n. Section 3 is devoted to the fault resiliency of HCN_n. Based on the fault tolerance of HCN_n, we derive its R^h-connectivities, i.e., $\kappa^1(HCN_n) = 2n$ and $\kappa^2(HCN_n) = 4n - 4$. Section 4 concentrates on the conditional diagnosability of HCN_n under the PMC model. Section 5 concludes the paper.

2. Preliminaries

2.1. Terminologies and notations

We use a graph $G = G(V, E)$ to represent an interconnection network, where a vertex $u \in V$ represents a processor and an edge $(u, v) \in E$ represents a link between vertices u and v. If at least one end of an edge is faulty, the edge is said to be faulty; otherwise, the edge is said to be fault-free. For a vertex u in G, $N(u)$ denotes the set of all neighbors of u, i.e., $N(u) = \{v \mid (u, v) \in E\}$. For a vertex subset $S \in V(G)$, we denote $N(S) = \bigcup_{u \in S} N(u) \setminus S$ the open neighborhood of S and $N[S] = N(S) \cup S$ the closed neighborhood of S. For brevity, $N(\{u, v\})$ and $N[\{u, v\}]$ are written as $N(u, v)$ and $N[u, v]$, respectively. The subgraph of G induced by S, denoted by $G[S]$, is the graph with the vertex-set S and the edge-set $\{(u, v) \mid (u, v) \in E(G), u, v \in S\}$. We use $d(u, v)$ to denote the distance between u and v, the length of a shortest path between u and v in G, and we also denote $d(u, G) = \min\{d(u, v) \mid v \in G\}$. The *diameter* of G is defined as the maximum distance between any two vertices in G. A *path* in a graph is a sequence of distinct vertices so that there is an edge joining consecutive vertices, with the length being the number of vertices in the sequence minus 1. A *cycle* is a path of length at least three where there is an edge joining the first and last vertices. A path (or cycle) of length k is called a k-path (or k-cycle).

For any subset $F \subset V$, the notation $G - F$ denotes a graph obtained by removing all vertices in F from G and deleting those edges with at least one end-vertex in F, simultaneously. If $G - F$ is disconnected, F is called a *separating set*. A separating set F is called a *k-separating set* if $|F| = k$. The maximal connected subgraphs of $G - F$ are called *components*. The *connectivity* $\kappa(G)$ of G is defined as the minimum k for which G has a k-separating set; otherwise $\kappa(G)$ is defined as $n - 1$ if $G = K_n$. A graph G is called to be *k-connected* if $\kappa(G) \geq k$.

The traditional connectivity $\kappa(G)$ of a network $G = G(V, E)$ is an important parameter to measure the fault tolerance of the network. However, there is an obvious deficiency in the definition of $\kappa(G)$, and it is tacitly assumed that all vertices adjacent to a vertex can potentially fail at the same time. To compensate for this shortcoming, it is natural to generalize the classical connectivity by introducing some conditions or restrictions on the separating set S and/or the components of $G - S$ [24]. To simplify the computation of $\kappa_R^h(G)$, Wan and Zhang [23] proposed a kind of conditional connectivity by placing some requirements on the components of $G - F$ only. Given a network based on a graph G and a positive integer h, the R^h-connectivity of G, denoted by $\kappa^h(G)$, is the minimum cardinality of a set of vertices in G, if any, whose deletion disconnects G, and every remaining component has minimum degree at least h. For the star graph S_n, Wan and Zhang [23] determined $\kappa^2(S_n) = 6(n-3)$ for $n \geq 4$. For the (n, k)-star graphs $S_{n,k}$ [35], Yang et al. [30] proved that $\kappa^1(S_{n,k}) = n + k - 3$, and $\kappa^2(S_{n,k}) = n + 2k - 5$ for $2 \leq k \leq n - 2$. For the Cayley graph $\Gamma_n(\Delta)$ generated by 2-trees, Cheng et al. [8] obtained that $\kappa^1(\Gamma_n(\Delta)) = 4n - 11$ and $\kappa^2(\Gamma_n(\Delta)) = 6n - 18$. These results generalize the corresponding parts of the popular alternating group graphs investigated by Zhang et al. [34].

2.2. Hierarchical cubic network HCN_n

Network reliability is one of the major factors in designing the topology of an interconnection network. Because of its elegant topological properties and the ability to emulate a wide variety of other frequently used networks, the hypercube has been one of the most popular interconnection networks for parallel computer/communication systems. However, when dealing with the parallel computers of very large scale, the port limitation due to the technology greatly affects the use of hypercube. The hierarchical cubic network, first proposed by Ghose and Desai [14], has almost half as many edges as a comparable hypercube. In other words, the degree of n-dimensional hierarchical cubic network HCN_n is almost half of that of a hypercube of the same size. Also importantly, the diameter of HCN_n is smaller than that of a hypercube of the same size [33].

An optimal shortest path routing algorithm in HCN_n is described by Yun and Park [32,33]. A node-to-set routing algorithm in HCN_n is first proposed by Chiang and Chen [7] and then improved by Fu et al. [12]. Recently, Bossard and Kaneko [2] have explored the node-to-set disjoint routing algorithm. Fu and Chen [11,13] investigate hamiltonicity and fault tolerant pancyclicity of HCN_n. Bossard [1] proposes an efficient algorithm generating in an HCN_n a decycling set of at most $2^{2n-1} - (2^{2n-2}/n + \lfloor 2^{n-1}/n \rfloor)$ nodes, which will protect from deadlocks, livelocks and starvations in resource allocation issues.